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Renormalization and the Equivalence Theorem: On-shell Scheme

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ABSTRACT

We perform an exhaustive analysis of the Equivalence Theorem both in the minimal Standard Model and in an Effective Electroweak Chiral Lagrangian up to $\mathcal{O}(p^4)$. We have considered the leading corrections to the usual prescription consisting in just replacing longitudinally polarized W or Z by the corresponding Goldstone bosons. The corrections appear through an overall constant multiplying the Goldstone boson amplitude as well as through additional diagrams. By including them we can extend the domain of applicability of the Equivalence Theorem, making it suitable for precision tests of the symmetry breaking sector of the Standard Model. The on-shell scheme has been used throughout. When considering the Equivalence Theorem in an Effective Chiral lagrangian we analyze its domain of applicability, as well as several side issues concerning gauge fixing, Ward identities, on-shell scheme and matching conditions in the effective theory. We have analyzed in detail the processes $W^+W^- \to W^+W^-$ and $W^+W^+ \to W^+W^+$ to illustrate the points made.

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1. Introduction

The Equivalence Theorem states that for any spontaneously broken gauge theory, provided the energy transfer is large enough, one can replace the longitudinal degrees of freedom of the massive vector bosons by the appropriate Goldstone bosons and use them to compute S-matrix elements.

Even though the Equivalence Theorem was proved originally [1-2] in the context of the minimal Standard Model, with a doublet of complex scalar fields, it has been realized[3-5] that it should remain valid for other theories exhibiting an equivalent set of fields and symmetries, even for non renormalizable ones.

This makes the Equivalence Theorem potentially very useful in investigations of the symmetry breaking sector of the Standard Model. Indeed, it has become customary to describe such a sector by a non-linear, non-renormalizable effective theory, the Effective Chiral Lagrangian[6]. The Goldstone bosons of the broken global symmetry $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ are collected in a matrix-valued dimensionless field $U(x)$. The operators in the Effective Chiral Lagrangian are classified according to the number of derivatives or gauge fields acting on $U(x)$. At low energies only the first terms in the expansion are of interest as the typical size of the expansion parameter in the minimal Standard Model is $p^2/(4\pi v)^2$ or p^2/M_H^2, whichever is larger. Since $v = 250$ GeV, if the Higgs is very heavy the expansion is clearly a very good one. On the other hand, if Nature has ruled that Higgs does not exist, we have to appeal to Technicolor or composite models to account for the breaking of the global symmetry and the appeareance of the Goldstone bosons. The coefficients of the Effective Chiral Lagrangian will then differ from the values they take in the minimal Standard Model. However, it would be unrealistic to expect dramatic changes in their order of magnitude. Thus it is not difficult to convince oneself that the contribution from the $\mathcal{O}(p^4)$ operators is all that it will be possible to measure in the near future (the contribution from the $\mathcal{O}(p^2)$ operators is universal and carries no information on whatever underlying physics gives the Z and W a mass).

Formally the non-linear, non-renormalizable Effective Chiral Lagrangian is very similar to the long distance effective description of strong interactions in terms of pions and kaons, the strong chiral lagrangian[7]. Much of what over the years has been learnt from the interactions of pions and kaons can then be easily taken over to the weak interaction case. For this purpose the Equivalence Theorem is instrumental.

In recent times there has been a flurry of papers dealing in one way or another with the Equivalence Theorem. The activity has proceeded mostly along two directions. First, it has been realized that the common textbook statement of the Equivalence Theorem, namely

$$A(W_L W_L \rightarrow W_L \ldots W_L) = (-i)^n A(\omega\omega \rightarrow \omega \ldots \omega) + \mathcal{O}(M/\sqrt{s}), \tag{1.1}$$

where ω denotes the goldstone boson 'eaten' by the appropriate W or Z boson, is not quite correct. For one thing, there is an overall factor $\mathcal{C}^n$ on the r.h.s of (1.1) [8-10]. The origin and relevance of this factor we will discuss in detail in the coming sections. Moreover it is incorrect to discard all the terms that in (1.1) are lumped together in the '$\mathcal{O}(M/\sqrt{s})$' bit. Except in the crudest of approximations both terms need to be kept. As we will show they both give subleading corrections that do not necessarily vanish in the large s limit and that are needed if one wants to perform a detailed comparison with the experimental

results. It should be stated right away that the Equivalence Theorem has been mostly used in the context of the so-called strongly interacting Higgs — the limit in which the quartic self coupling in the scalar sector of the minimal Standard Model becomes large. In this limit the dominant contributions are correctly taken into account by (1.1). Yet, the corrections to (1.1) are not negligible at all. This is the origin of some misgivings that have been raised[11-13] concerning the usefulness of the Equivalence Theorem. After studying this issue in detail, we hope to convince the reader that the Equivalence Theorem remains a powerful tool to disentangle the scalar sector of the Standard Model.

Second, we all know that the minimal Standard Model need not be the correct theory for the symmetry breaking sector. We know that the Higgs is particularly well hidden in the Electroweak Theory[14]. It makes a lot of sense to try and investigate possible departures from the minimal Standard Model by setting bounds on the $\mathcal{O}(p^4)$ coefficients of the Effective Chiral Lagrangian. Scattering of longitudinal W's and Z's are amongst the clearest ways of doing this and the Equivalence Theorem comes handy. As we have mentioned, it should remain valid in effective, non-renormalizable theories. However, these have a limited range of validity as we scale up the energy (the upper bound being $4\pi v$ or Λ, Λ being the mass of the first resonance in the strongly interacting scalar sector, whichever is smallest), so the energy cannot be too large. It cannot be too small either on account of the uncalculated pieces on the r.h.s of (1.1), so the range of validity seems to be limited[13]. We will show that taking properly into account the next to leading corrections to the Equivalence Theorem improves considerably the situation and allows for practical applications with the required level of precision.

The accuracy reached in many experiments testing the Electroweak Theory is such that radiative corrections have necessarily to be taken into account. Nowadays there seems to exist ample consensus in choosing the on-shell scheme to carry out the renormalization program[15-16]. For this reason we have elected to work within this scheme in our discussion of the Equivalence Theorem as is conceptually simple and technically convenient. To our knowledge this is the first time that such an analysis is presented.

In deriving the above results we have been led to a number of colateral issues. We believe that some of the results are interesting in their own right and we have made an effort to collect them either in the main body of the paper or in the appendices. Amongst these we should mention: a discussion of the renormalization in the longitudinal sector in the on-shell scheme, Ward identities in the non-linear realization, modifications to the on-shell scheme when the Higgs is not present, and the full lagrangian expanded up to terms with four fields in the non-linear variables. To keep the discussion simple we have restricted ourselves to the charged sector. No conceptually new issues appear in the neutral sector, but the $\gamma - Z$ mixing complicates the analysis considerably.

Let us discuss briefly the way the paper is organized. In section 2 we review some Ward identities in the minimal Standard Model and discuss the formulation of the Electroweak Theory in terms of the non-linear variables suitable in the large Higgs mass limit. Section 3 is devoted to the derivation of the Equivalence Theorem and the $\mathcal{C}$ factor in the minimal Standard Model. We then proceed to apply, in section 4, the previous results to the analysis of the processes $W^+W^- \to W^+W^-$ and $W^+W^+ \to W^+W^+$ in the minimal Standard Model. The gauge-fixing procedure, Ward identities and the extension of the

Equivalence Theorem to the Effective Chiral Lagrangian is discussed in sections 5 and 6. We have also included in section 5 a discussion on the matching conditions and how the results of the minimal Standard Model are reproduced by a particular choice of the $\mathcal{O}(p^4)$ coefficients in the Effective Chiral Lagrangian. In section 7 we apply the Equivalence Theorem to the process $W^+W^- \to W^+W^-$ in the Effective Chiral Lagrangian, including a discussion on the domain of applicability. Finally, our conclusions are summarized in section 8.

2. Gauge Fixing and Ward Identities in the Minimal Standard Model

We shall start our discussion by outlining the derivation of some Ward identities in the minimal Standard Model involving the longitudinal sector of the theory. We will first analyze the Standard Model in the usual linear variables and move afterwards to the non-linear representation, more suitable to deal with a strongly interacting scalar sector. Throughout this paper we will work in the on-shell scheme renormalization scheme and we shall basically adhere to the conventions of [16] and [17-19], except for a field redefinition in the scalar sector.

2.1 Linear Realization

The minimal Standard Model lagrangian with a doublet of complex fields Φ is

$$\mathcal{L}_{SM} = D_\mu \Phi^\dagger D^\mu \Phi - \lambda |\Phi|^4 + \mu^2 |\Phi|^2 + \mathcal{L}_{YM} + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \tag{2.1.1}$$

with $D_\mu = \partial_\mu + \frac{1}{2} ig W^i_\mu \sigma^i + \frac{1}{2} ig' B_\mu$. The Higgs doublet is

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \omega^2 + i\omega^1 \\ \sigma - i\omega^3 \end{pmatrix}. \tag{2.1.2}$$

In the notation of [16] $\omega^2 \to \varphi^1$, $\omega^1 \to -\varphi^2$ and $\omega^3 \to -\chi$. The gauge fixing and Faddeev-Popov terms are

$$\mathcal{L}_{GF} = -\frac{1}{2}(2F^+F^- + F^3F^3 + F^0F^0), \qquad \mathcal{L}_{FP} = \sum_{\alpha,\beta=i,0} \bar{c}^\alpha \frac{\delta F^\alpha}{\delta \theta^\beta} c^\beta, \tag{2.1.3}$$

with $i = 1,2,3$ running over $SU(2)_L$ indices and $\alpha = 0$ corresponding to the $U(1)_Y$ part

$$\begin{aligned} F^\pm &= \frac{1}{\sqrt{\xi_1^W}} \partial^\mu W^\pm_\mu - M_W \sqrt{\xi_2^W}\, \omega^\pm, \\ F^3 &= \frac{1}{\sqrt{\xi_1^3}} \partial^\mu W^3_\mu - M_W \sqrt{\xi_2^3}\, \omega^3, \\ F^0 &= \frac{1}{\sqrt{\xi_1^B}} \partial_\mu B^\mu + (M_Z^2 - M_W^2)^{\frac{1}{2}} \sqrt{\xi_2^B}\, \omega^3. \end{aligned} \tag{2.1.4}$$

The charged fields $W^\pm_\mu$ and $\omega^\pm$ are defined by $W^\pm_\mu = (W^1_\mu \mp iW^2_\mu)/\sqrt{2}$ and $\omega^\pm = (\omega^1 \mp i\omega^2)/\sqrt{2}$. In this work we will not consider neutral fields and we will drop the indices 'W'

and 'B', being understood that unless stated otherwise we will be referring to the charged fields. Notice that in the on-shell scheme ξ_1 and ξ_2, although equal classically, have to be kept different beyond tree level because they renormalize differently. Furthermore, the introduction of two gauge parameters allows for the elimination of the divergences appearing in mixed $W-\omega$ Green functions. This is discussed in Appendix A.

The starting point in our discussion is the generating functional of connected Green functions

$$Z[J,\eta,\bar{\eta}] = \int \mathcal{D}X\mathcal{D}\bar{c}\mathcal{D}c\,\exp\{i\int d^4x[\mathcal{L}_{SM} + X^\alpha J^\alpha + \bar{c}^\alpha\eta^\alpha + \bar{\eta}^\alpha c^\alpha]\}. \tag{2.1.5}$$

(X^α collectively denotes the fields in the theory, and $\bar{c}$, c and η, $\bar{\eta}$ are the ghosts and their sources, respectively). A summation over space-time points as well as external indices is understood. By differentiating twice and setting the sources equal to zero we obtain the bare propagators of the theory. In the longitudinal sector we have gauge, Goldstone boson and mixed propagators. Their decomposition in terms of invariant functions is

$$\begin{aligned}
D^{W^+W^-}_{\mu\nu}(k) &= (-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2})\Lambda_T^{W^+W^-}(k^2) - \frac{k_\mu k_\nu}{k^2}\Lambda_L^{W^+W^-}(k^2),\\
D^{W^+\omega^-}_{\mu}(k) &= ik_\mu\Lambda^{W^+\pi^-}(k^2),\\
D^{\omega^+\omega^-}(k) &= \Lambda^{\pi^+\pi^-}(k^2).
\end{aligned} \tag{2.1.6}$$

In terms of self-energies

$$\begin{aligned}
\Lambda_T^{W^+W^-} &= \frac{i}{k^2 - M_0^2 + \Sigma_T},\\
\Lambda_L^{W^+W^-} &= \frac{i\xi_1^0}{k^2 - \xi_1^0 M_0^2 + \xi_1^0\Sigma_L},\\
\Lambda^{\omega^+\omega^-} &= \frac{i}{k^2 - \xi_2^0 M_0^2 + \Sigma_\omega},\\
\Lambda^{W^+\omega^-} &= \frac{i\xi_1^0}{k^2 - M_0^2\xi_1^0}\Sigma_{W\omega}\frac{1}{k^2 - M^2\xi_2^0}.
\end{aligned} \tag{2.1.7}$$

The last expression is valid at the one-loop level only. We will restrict ourselves to this order as going beyond this in the Standard Model is only of academic interest at present. Since we will be dealing exclusively with charged W's, in order not to unnecessarily clutter our formulae we have suppresed the indices in masses and self-energies whenever no confussion is possible. The relation between bare (Σ) and renormalized ($\hat{\Sigma}$) self-energies is given in Appendix A. For the mixed propagator we have only considered the one-loop expression in (2.1.7) because this is all we will need in what follows. Since we work in a 't Hooft gauge there is no tree level contribution to $\Lambda^{W^+\omega^-}$. Furthermore note that with our conventions $\Lambda^{W^+\omega^-} = -\Lambda^{\omega^+W^-} = \Lambda^{W^-\omega^+}$ on account of the hermiticity of the effective action.

When the sources are set to zero the generating functional (2.1.5) is invariant under the BRS transformation ($\zeta^2 = 0$)

$$\begin{aligned}
\delta W^i_\mu &= (-\delta^{ik}\partial_\mu + g\epsilon^{ijk}W^j_\mu)c^k\zeta, \\
\delta B_\mu &= -\partial_\mu c^0\zeta, \\
\delta\omega^i &= \frac{g}{2}(\sigma\delta^{ik} + \epsilon^{ijk}\omega^j)c^k\zeta - \frac{g'}{2}(\sigma\delta^{i3} - \epsilon^{ij3}\omega^j)c^0\zeta, \\
\delta\sigma &= -\frac{g}{2}\omega^i c^i\zeta + \frac{g'}{2}\omega^3 c^0\zeta, \\
\delta\bar{c}^\alpha &= F^\alpha\zeta, \\
\delta c^i &= -\frac{1}{2}\epsilon^{ijk}c^j c^k\zeta, \\
\delta c^0 &= 0.
\end{aligned} \tag{2.1.8}$$

The fact that the gauge group is $SU(2)_L \times U(1)_Y$ makes the BRS transformation somewhat involved. Using the invariance of the action we arrive at

$$\langle 0|J^\gamma\delta X^\gamma + \delta\bar{c}^\gamma\eta^\gamma + \bar{\eta}^\gamma\delta c^\gamma|0\rangle_{J,\eta,\bar{\eta}} = 0. \tag{2.1.9}$$

A further derivation w.r.t. η followed by the limit $\eta = \bar{\eta} = 0$ gives

$$\langle 0|F^\beta(X(y))\zeta|0\rangle_J = \langle 0|\delta\bar{c}^\beta(y)|0\rangle_J = -i\langle 0|J^\gamma\delta X^\gamma\bar{c}^\beta(y)|0\rangle_J. \tag{2.1.10}$$

Acting now with $F^\alpha(\frac{\delta}{i\delta J(x)})$ on both sides of (2.1.10), using that the gauge fixing is *linear* in the fields and the fact that

$$F^\alpha(\delta X(x)) = (\frac{\delta F^\alpha}{\delta\theta^\sigma}c^\sigma)(x)\zeta, \tag{2.1.11}$$

we get[16]

$$F^\alpha(\frac{\delta}{i\delta J(x)})F^\beta(\frac{\delta}{i\delta J(y)})Z[J]|_{J=0} = i\delta^{\alpha\beta}\delta(x-y)Z[0]. \tag{2.1.12}$$

To be specific let us concentrate in the gauge fixing condition for the charged fields, $F^\pm$. Eq. (2.1.12) can be easily written in terms of propagators. Using (2.1.6)

$$k^2\Lambda_L^{W^+W^-} - 2M_0k^2\sqrt{\xi_1^0\xi_2^0}\Lambda^{W^+\omega^-} - \xi_1^0\xi_2^0M_0^2\Lambda^{\omega^+\omega^-} = i\xi_1^0. \tag{2.1.13}$$

We have used that $\Lambda^{W^+\omega^-} = -\Lambda^{\omega^+W^-}$. Now we substitute (2.1.7) into (2.1.13). If we work at tree level we can set $\xi_1^0 = \xi_2^0 = \xi$ and (2.1.13) is just an identity. At the next order we have to keep track of the self-energies, which are $\mathcal{O}(g^2)$ and of the difference between ξ_1^0 and ξ_2^0 creeping in from the tree level expression. Then one finally gets with one-loop precision

$$k^2\xi_1^0\Sigma_L - M_0^2\Sigma_\omega\xi_2^0 + 2M_0k^2\sqrt{\xi_1^0\xi_2^0}\Sigma_{W\omega} = 0 \tag{2.1.14}$$

These Ward identities will be useful on two counts. On the one hand they allow to express the mixed propagator and self-energy in terms of the W and ω propagators and self-energies. Moreover they provide important relations between the different renormalization constants in the longitudinal sector. The relation between the bare and renormalized expressions is obtained through the use of the renormalization constants described in Appendix A. We shall demand that the renormalized gauge parameters are equal, i.e. $\xi_1 = \xi_2$. Since $Z_{\xi_1} \neq Z_{\xi_2}$ this requires $\xi_1^0 \neq \xi_2^0$. As a consequence there is a net counterterm for the self-energy $\Sigma_{W\omega}(k^2)$ beyond tree level. (Recall that in t'Hooft gauges the mixed $W-\omega$ piece cancels off at tree level between the gauge gauge fixing term and the kinetic piece $(D_\mu\Phi)^\dagger D^\mu\Phi$ once the symmetry is broken, $\langle\sigma\rangle = v$, and we shift $\sigma \to v+\sigma$.)

In terms of renormalized quantities (2.1.14) reads

$$k^2(\hat{\Sigma}_L + 2M\hat{\Sigma}_{W\omega}) - M^2\hat{\Sigma}_\omega = (k^2-\xi M^2)(\frac{k^2}{\xi}(\delta Z_W - \delta Z_{\xi_1}) - M^2(\delta Z_\omega + \delta Z_M + \delta Z_{\xi_2})). \tag{2.1.15}$$

Notice that (2.1.15) provides us with some combinations of renormalization constants which are necessarily finite, such as $Z_W Z_{\xi_1}^{-1}$ and $Z_\omega Z_M Z_{\xi_2}$.

We could have used —in principle— a gauge condition other than (2.1.4). This would have not affected the physical S-matrix elements, but would have changed the formulation of the Ward identities needed to prove the Equivalence Theorem and, of course, reshuffled different contributions amongst different diagrams. Some of the difficulties encountered in other gauges will we commented upon next.

2.2. Non-linear Realization

If the Higgs mass is large, i.e. if the quartic coupling λ is large, the lagrangian (2.1.1) is not written in the most convenient set of variables. Indeed, the quartic coupling affects all four scalar fields (σ, ω^i), thus involving the Goldstone bosons of the $SU(2)_L \times SU(2)_R \to SU(2)_V$ breaking which mix with the longitudinal W's and Z. On the other hand, since $M_H^2 = 2\lambda v^2$, the internal exchange of the Higgs boson is strongly suppressed. These two facts together lead to a tremendous amount of cancellation between different diagrams, a fact well know to any practitioner of the linear sigma model[20].

It is far more convenient to rewrite the scalar sector of the Standard Model using a non-linear realization. The way to proceed is to introduce the matrix-valued field $M(x)$

$$M(x) = \sqrt{2}(\,\tilde{\Phi}\Phi\,), \tag{2.2.1}$$

where $\tilde{\Phi}$ is the hypercharge conjugated doublet. We then perform the change of variables

$$M = \rho U \qquad U = \exp\frac{i}{v}\pi^i\sigma^i, \tag{2.2.2}$$

with $U \in SU(2)_L \times SU(2)_R/SU(2)_V$. The unitary matrix U collects the Goldstone bosons of the broken global symmetry π^i. The fields π^i are related by a non-linear tranformation (involving the ρ field) to the ω^i used in the previous subsection. Since M transforms linearly, so does U, but the Goldstone bosons themselves transform non-linearly

$$U'(x) = e^{\frac{i}{2}\alpha^i(x)\sigma^i}U(x)e^{-\frac{i}{2}\alpha^0(x)\sigma^3}, \tag{2.2.3}$$

$$\delta\pi^i = \frac{1}{2}\left(v(\alpha^i - \alpha^0\delta^{i3}) - \epsilon^{ijk}\alpha^j\pi^k + \alpha^0(\pi^2\delta^{i1} - \pi^1\delta^{i2})\right) + \frac{1}{6v}\pi^j\pi^l\left(\alpha^k(\delta^{li}\delta^{kj} - \delta^{jl}\delta^{ki}) - \alpha^0(\delta^{j3}\delta^{li} - \delta^{i3}\delta^{jl})\right) + \dots \tag{2.2.4}$$

The field ρ is inert under the gauge group. This is in contradistinction to what happens to the σ field in a linear realization. The field ρ gets a v.e.v. when the symmetry is broken and, as usual, we shift $\rho \to v + \rho$. The steps required in going from the linear to the non-linear realization have been discussed in detail in [19].

The lagrangian in the non-linear realization is

$$\mathcal{L}_{SM} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho - \rho\lambda v(v^2 + \frac{\mu^2}{\lambda}) - \frac{1}{2}\rho^2(\mu^2 + 3v^2\lambda) - \lambda v\rho^3 - \frac{1}{4}\lambda\rho^4 + \frac{1}{4}(\rho+v)^2\mathrm{Tr}D_\mu U^\dagger D^\mu U + \mathcal{L}_{GF} + \mathcal{L}_{FP}. \tag{2.2.5}$$

with $D_\mu U = \partial_\mu U + \frac{1}{2}igW^i_\mu\sigma^i U(x) - \frac{1}{2}ig' B_\mu U(x)\sigma^3$. For the gauge-fixing part one could just use the transformed of (2.1.4)

$$\begin{aligned}\mathcal{L}_{GF} \equiv & -\frac{1}{2}\sum_{1=1,3} F^i F^i - \frac{1}{2}F^0F^0 \\ = & -\frac{1}{2\xi^W}\sum_{i=1,3}(\partial^\mu W^i_\mu + \frac{i}{4}gv\xi^W(\rho+v)\mathrm{Tr}\tau^i U)^2 \\ & - \frac{1}{2\xi^B}(\partial^\mu B_\mu - \frac{i}{4}g'v\xi^B(\rho+v)\mathrm{Tr}\tau^3 U)^2.\end{aligned} \tag{2.2.6}$$

(We shall not distinguish here between ξ_1 and ξ_2 to keep our formulae manageable). Finally

$$\begin{aligned}\mathcal{L}_{FP} = & \partial^\mu c^{0\dagger}\partial_\mu c^0 + \partial^\mu c^{i\dagger}\partial_\mu c^i + g\, c^{i\dagger}(\partial^\mu W^k_\mu\epsilon^{ikj} - \frac{1}{8}gv\xi^W(\rho+v)\mathrm{Tr}\tau^i\tau^j U)c^j \\ & - \frac{1}{8}g'^2 v\xi^B c^{0\dagger}(\rho+v)\mathrm{Tr}Uc^0 + \frac{1}{8}\sqrt{gg'}\,v\,(g'\xi^B c^{0\dagger}c^i + g\xi^W c^{i\dagger}c^0)(\rho+v)\mathrm{Tr}\tau^3\tau^i U.\end{aligned} \tag{2.2.7}$$

In Landau gauge ($\xi = 0$) ghosts decouple from Goldstone bosons.

$\mathcal{L}_{SM} + \mathcal{L}_{GF} + \mathcal{L}_{FP}$ is invariant under BRS tranformations. The gauge fields transform as in (2.1.8) and

$$\begin{aligned}\delta\rho &= 0, \\ \delta U &= \frac{i}{2}g\sigma^i U c^i\zeta - \frac{i}{2}g' U\sigma^3 c^0\zeta, \\ \delta\bar{c}^\alpha &= F^\alpha\zeta, \\ \delta c^i &= -\frac{1}{2}\epsilon^{ijk}c^j c^k\zeta, \\ \delta c^0 &= 0.\end{aligned} \tag{2.2.8}$$

Since F^α is linear in the U field (but not in the π field), a Ward identity similar to (2.1.13) can be derived

$$k^2\Lambda_L^{W^+W^-} + ik^2 M_0\sqrt{\xi_1^0\xi_2^0}\tilde{\Lambda}^{W^+\pi^-} + \frac{M_0^2}{4}\xi_1^0\xi_2^0\tilde{\Lambda}^{\pi^+\pi^-} = i\xi_1^0. \tag{2.2.9}$$

(We have restored the two different gauge parameters ξ_1^0, ξ_2^0.) The tilded Green functions in the previous equation are defined as

$$ik_\mu \tilde{\Lambda}^{W^+\pi^-}(k^2) = \int d^4x e^{-ikx} \langle 0|W_\mu^+(x)[(v+\rho)U^-](0)|0\rangle, \tag{2.2.10}$$

$$\tilde{\Lambda}^{\pi^+\pi^-}(k^2) = \int d^4x e^{-ikx} \langle 0|[(v+\rho)U^+](x)[(v+\rho)U^-](0)|0\rangle, \tag{2.2.11}$$

with $U^\pm = \mathrm{Tr}\sigma^\pm U$. The Ward identity (2.2.11) thus actually relates an infinite number of Green functions when written in terms of the π fields. These Ward identities can be expanded in inverse powers of v and solved iteratively. In practice, however, it is more useful to shift to another set of gauge conditions which are linear in the non-linear Goldstone fields π. Let us take instead

$$\begin{aligned}
F^\pm &= \frac{1}{\sqrt{\xi_1^W}} \partial^\mu W_\mu^\pm - M_W \sqrt{\xi_2^W} \pi^\pm, \\
F^3 &= \frac{1}{\sqrt{\xi_1^3}} \partial^\mu W_\mu^3 - M_W \sqrt{\xi_2^3}\, \pi^3, \\
F^0 &= \frac{1}{\sqrt{\xi_1^B}} \partial_\mu B^\mu + (M_Z^2 - M_W^2)^{\frac{1}{2}} \sqrt{\xi_2^B}\, \pi^3.
\end{aligned} \tag{2.2.12}$$

Then the Ward identities (2.1.13) and (2.1.14) remain strictly valid with

$$ik_\mu \Lambda^{W^+\pi^-}(k^2) = \int d^4x e^{-ikx} \langle 0|W_\mu^+(x)\pi^-(0)|0\rangle, \tag{2.2.13}$$

$$\Lambda^{\pi^+\pi^-}(k^2) = \int d^4x e^{-ikx} \langle 0|\pi^+(x)\pi^-(0)|0\rangle. \tag{2.2.14}$$

Even though in this gauge the expressions are formally the same, the bare Green functions themselves that appear in these expressions differ numerically from those obtained in the linear realization. The equality between both realizations is only guaranteed at the level of S-matrix elements[21], or for connected Green functions involving only gauge fields[19] (since they have not changed in passing from linear to non-linear variables).

The lagrangian in this gauge, expanded up to four fields, is given in Appendix D. Notice that the Goldstone bosons have only derivative couplings and that the coupling λ now affects only the ρ field. If the mass of this radial excitation, which is to be identified with M_H, is very large it makes sense to go one step further and integrate ρ out obtaining an effective lagrangian that will reproduce the Standard Model at energies much below M_H. This will be discussed in section 5 and 6. For the time being let us return to the Standard Model in the linear realization.

3. The Equivalence Theorem in the Standard Model

We will start by restricting ourselves to a physical process that contains only one external longitudinal vector boson and towards the end of the section we will consider the generalization to an arbitrary number of vector bosons.

Let us begin by recalling some elementary facts in the Standard Model. The first point to remember is that the gauge conditions (2.1.3) must be satisfied by the *in* and *out* states. Therefore (and restricting ourselves to the charged boson case)

$$\langle 0|F^{\pm}|\psi\rangle = 0, \tag{3.1}$$

$F^{\pm}$ being the gauge condition (2.1.4) and $|\psi\rangle$ some physical state. The second point to remember is that the condition (3.1) does not determine the state completely. We shall use a 2-vector notation to represent the W_μ and ω contents of an external state. Thus a *in* state represented by

$$(\epsilon^\mu, 0) \qquad k\cdot\epsilon = 0, \tag{3.2}$$

meaning that the asymptotic field representing the Goldstone boson is zero, and one represented by

$$(\epsilon^\mu - \frac{k^\mu}{M}\theta, \frac{i}{\sqrt{\xi_1\xi_2}}\theta), \tag{3.3}$$

where the asymptotic field representing the Goldstone boson is multiplied by the second entry of the vector (3.3), fulfill the same gauge condition; they are the same physical state[22]. Since $k^2 = M^2$, for massive vector bosons it is not possible to take $\epsilon_L^\mu \propto k^\mu$. Rather the polarization vectors associated to longitudinal W's are of the form

$$\epsilon_L^\mu = \frac{k^\mu}{M} + v^\mu. \tag{3.4}$$

Therefore states that asymptotically correspond to longitudinally polarized W's , described by polarization vectors ϵ_L^μ, cannot be completely gauged away and traded for Goldstone bosons. The best one can do is to single out within the equivalence class corresponding to a given physical state $|\psi\rangle$ two extreme cases. One is just (3.2), the other is obtained by gauging away the k^μ part in (3.4), namely

$$(v^\mu, \frac{i}{\sqrt{\xi_1\xi_2}}). \tag{3.5}$$

Finally, a third point to remember is that at very high energies the v^μ part is unimportant. Let us write explicitly the longitudinal polarization vector for a W particle moving forward in the z-axis with momentum p and energy E

$$\epsilon_L^\mu = (\frac{p}{M}, 0, 0, \frac{E}{M}). \tag{3.6}$$

Therefore $\epsilon_L^\mu - k^\mu/M$ is of $\mathcal{O}(M/E)$, eventually negligible when compared to the first term in (3.4) which is of $\mathcal{O}(E/M)$. Also, and for the same reason, at high energies the scattering of longitudinal components of vector bosons dominates over transverse ones[2,20,23].

In a way eqs. (3.2) through (3.5) already encompass the Equivalence Theorem. At high energies, the scattering of Goldstone bosons has something to do with the scattering of longitudinally polarized W's. Obviously, this is only a qualitative statement and one

needs to go beyond this to make detailed calculations. For instance, the Equivalence Theorem appears to be closely linked to the 't Hooft-Feynman gauge[24], in which one sets $\xi_1 = \xi_2 = 1$. What happens then for, say, Landau gauge where Goldstone boson physics is more manifest? Can (1.1) be derived then? On the other hand, Green functions involving unphysical states (such as those constructed with only ω fields) need not be gauge invariant. Only keeping the contribution from the v^μ part ensures that and it is necessary to be well aware of this fact.

The answer to the first objection is of course that eqs. (3.2)-(3.5) are formal relations valid for bare quantities while we are interested in relations involving S-matrix elements. Furthermore as the fields propagate they mix among themselves since they have the same quantum numbers. Let us start by deriving a relation involving asymptotic fields. We shall follow the approach of [9], but we will deviate at some point in order not to introduce unphysical Green functions involving ghosts. The condition (3.1) reads (for a negatively charged *in* state $|\psi\rangle$)

$$\begin{aligned} 0 &= \int d^4x e^{-ikx} \langle 0|\partial^\mu W_\mu^+(x) - M_0\sqrt{\xi_1^0\xi_2^0}\omega^+(x)|\psi\rangle \\ &= (ik^\mu, -M_0\sqrt{\xi_1^0\xi_2^0}) \int d^4x e^{-ikx} \langle 0|(W_\mu^+(x), \omega^+(x))|\psi\rangle^\top|_{k^2=M^2}. \end{aligned} \tag{3.7}$$

All quantities and fields appearing above are understood to be bare ones. Using now the reduction formula (see e.g. [25]) to relate Green functions to amplitudes we get

$$K^M D_{MN}\langle W^N|\psi\rangle^\top|_{k^2=M^2} = 0, \tag{3.8}$$

where $K^M = (ik^\mu, -M_0\sqrt{\xi_1^0\xi_2^0})$,

$$D_{MN} = \begin{pmatrix} D_{\mu\nu}^{W^+W^-}(k) & D_\mu^{W^+\omega^-}(k) \\ D_\mu^{\omega^+W^-}(k) & D^{\omega^+\omega^-}(k) \end{pmatrix}, \tag{3.9}$$

and

$$\langle W^N(k)|\psi\rangle = \langle (W^{-\nu}(k), \omega^-(k)|\psi\rangle. \tag{3.10}$$

(3.10) is not quite an amplitude because is not yet contracted with ϵ_ν, but an amputated Green function. Everywhere we understand that the limit $k^2 \to M^2$ has to be taken.

A straightforward calculation that makes use of the Ward identity (2.1.13) leads to

$$ik^\nu\langle W_\nu^-(k)|\psi\rangle = \frac{k^2}{M_0\sqrt{\xi_1^0\xi_2^0}} \frac{i\xi_1^0 - k^2\Lambda_L^{W^+W^-} - M_0^2\xi_1^0\xi_2^0\Lambda^{\omega^+\omega^-}}{i\xi_1^0 + k^2\Lambda_L^{W^+W^-} + M_0^2\xi_1^0\xi_2^0\Lambda^{\omega^+\omega^-}}\langle\omega^-(k)|\psi\rangle. \tag{3.11}$$

We have to set $k^2 = M^2$ in the above expression. All quantities and fields are still unrenormalized. We now proceed to write everything in terms of renormalized quantities using the renormalization constants described in Appendix A. The external legs require special treatment. In the on-shell scheme one commonly uses a minimal set of renormalization constants that make finite all Green functions, but which do not guarantee a unit residue

for the vector boson propagators[16,26] (other that the photon). To fix this problem one uses for the external legs $\tilde{Z}_W$ and $\tilde{Z}_\omega$ defined in (A.4).

Introducing the renormalization constants into (3.11) one finally gets a relation between renormalized amputated Green functions

$$ik^\nu \langle W_\nu^-(k)|\psi\rangle = M\mathcal{C}\langle\omega^-(k)|\psi\rangle. \tag{3.12}$$

$\mathcal{C}$ is given by

$$\mathcal{C} = \left(\frac{\tilde{Z}_W}{\tilde{Z}_\omega}\right)^{1/2} \frac{k^2}{M^2}\frac{1}{Z\sqrt{\xi_1\xi_2}}\frac{i\xi_1 - k^2 Z_W Z_{\xi_1}^{-1}\hat{\Lambda}_L^{W^+W^-} - \xi_1\xi_2 M^2 Z Z_\omega Z_{\xi_1}^{-1}\hat{\Lambda}^{\omega^+\omega^-}}{i\xi_1 + k^2 Z_W Z_{\xi_1}^{-1}\hat{\Lambda}_L^{W^+W^-} + \xi_1\xi_2 M^2 Z Z_\omega Z_{\xi_1}^{-1}\hat{\Lambda}^{\omega^+\omega^-}} \tag{3.13}$$

As usual, the renormalized propagators appearing in (3.13) are to be evaluated at $k^2 = M^2$. In the above expression $Z = (Z_M Z_{\xi_1} Z_{\xi_2})^{1/2}$. The renormalization constants that show up in (3.13) appear in combinations so as to make $\mathcal{C}$ finite. This can be easily checked by recalling (2.1.15). Notice that the renormalization of the external legs $\tilde{Z}_W$ and $\tilde{Z}_\omega$ have been included in $\mathcal{C}$. The presence of this factor in the Equivalence Theorem has been detected before[8-10]. The expression given here is new, however. In [9,10] $\mathcal{C}$ is given in terms of Green functions involving ghosts.

The factor $\mathcal{C}$ as given above is valid to all orders. In the on-shell scheme and at the one loop level all renormalization constants are expressable in terms of bare self energies. If we particularize to this order $\mathcal{C}$ becomes

$$\mathcal{C} = \left(1 + \frac{1}{2}(\delta\tilde{Z}_W - \delta\tilde{Z}_\omega) + \frac{1}{2M^2}\left(\Sigma_T(M^2) - \Sigma_L(M^2) - \Sigma_\omega(M^2)\right)\right). \tag{3.14}$$

It is remarkable the simplicity of this expression, which is valid in any gauge (although the bare self energies themselves do depend on the gauge). Furthermore the leading contribution is just 1, reproducing the naive arguments at the beginning of this section using Feynman gauge. Let's now analyze the properties of eq.(3.14). Obviously $\mathcal{C}$ is finite in the Standard Model. It turns out to be independent of M_H too (see the Σ's, $\tilde{Z}$'s in Appendix A). At least at the one loop level in the on-shell scheme, the $\mathcal{C}$ factor, and the Equivalence Theorem by extension, do not have any sizeable corrections due to the scalar sector and only contributions of $\mathcal{O}(g^2)$ appear.

As it stands, (3.12) relates finite, but yet unphysical, quantities. We rather write it as

$$\epsilon^\mu \langle W_\mu^-(k)|\psi\rangle = -i\mathcal{C}\langle\omega^-(k)|\psi\rangle + v^\mu\langle W_\mu^-(k)|\psi\rangle, \tag{3.15}$$

which we denote as

$$\langle W_L^-|\psi\rangle = -i\mathcal{C}\langle\omega^-|\psi\rangle + \langle\tilde{W}^-|\psi\rangle. \tag{3.16}$$

The l.h.s. is now a physical S-matrix amplitude. The r.h.s. is a sum of two pieces neither of which is physical. Both are gauge dependent, but the gauge dependence cancels between them and also thanks to the $\mathcal{C}$ factor (more on this will be discussed later). All matrix elements are to be computed at $k^2 = M^2$. Since the Goldstone boson mass in the

Standard Model is a function of the gauge parameter except in Feynman gauge the first matrix element on the r.h.s. is also off-shell in general.

It is a simple matter to extend the above results to several external longitudinally polarized W's. We proceed iteratively, applying the above procedure one by one to the external W's taking into account that the vector K^M for an outgoing W^+ or W^- with momentum k^μ is $K^M = (ik^\mu, -M\sqrt{\xi_1\xi_2})$, while for an incoming W^+ or W^- with momentum k^μ is $K^M = (-ik^\mu, -M\sqrt{\xi_1\xi_2})$. We then end up with

$$
\begin{aligned}
\langle W_L W_L W_L \ldots | \psi \rangle =& (-i)^n \mathcal{C}^n \langle \omega\omega\omega \ldots | \psi \rangle \\
&+ (-i)^{n-1} \mathcal{C}^{n-1} \langle \tilde{W}\omega\omega \ldots | \psi \rangle + (-i)^{n-1} \mathcal{C}^{n-1} \langle \omega\tilde{W}\omega \ldots | \psi \rangle \ldots \\
&+ \mathcal{O}((v^\mu)^2).
\end{aligned} \tag{3.17}
$$

For incoming W_L's we have to replace the appropriate $-i$ by a $+i$. For instance

$$
\begin{aligned}
A(W_L^+ W_L^- \to W_L^+ W_L^-) =& \mathcal{C}^4 A(\omega^+\omega^- \to \omega^+\omega^-) \\
&+ i\mathcal{C}^3 A(\tilde{W}^+\omega^- \to \omega^+\omega^-) + i\mathcal{C}^3 A(\omega^+\tilde{W}^- \to \omega^+\omega^-) \\
&- i\mathcal{C}^3 A(\omega^+\omega^- \to \tilde{W}^+\omega^-) - i\mathcal{C}^3 A(\omega^+\omega^- \to \omega^+\tilde{W}^-) \\
&+ \mathcal{O}((v^\mu)^2).
\end{aligned} \tag{3.18}
$$

We shall lump together the four pieces with one W_μ external field contracted with v^μ and three ω's under the symbol $A(\tilde{W}\omega\omega\omega)$

$$
\begin{aligned}
A(\tilde{W}www) =& i\mathcal{C}^3 A(\tilde{W}^+\omega^- \to \omega^+\omega^-) + i\mathcal{C}^3 A(\omega^+\tilde{W}^- \to \omega^+\omega^-) \\
&- i\mathcal{C}^3 A(\omega^+\omega^- \to \tilde{W}^+\omega^-) - i\mathcal{C}^3 A(\omega^+\omega^- \to \omega^+\tilde{W}^-).
\end{aligned} \tag{3.19}
$$

Of course, if the Equivalence Theorem is ever going to be useful we must be able to stop the expansion in (3.18) at some, preferably early, point. How far we need to go depends both the energy of the process (since further contributions are suppressed by additional powers of the energy) and on the precision required (are we merely interested in the limit where λ is large, or $\mathcal{O}(g^2)$ corrections need to be taken into account?).

We just saw that v^μ is suppressed by a factor M^2/E^2 with respect to ϵ_L^μ. The additional terms, $A(\tilde{W}\omega\omega\omega)$ are therefore suppressed by this same factor with respect to $A(\omega\omega\omega\omega)$. In a renormalizable theory the latter behaves for large E as a constant, at worst. Thus the additional term should indeed be of $\mathcal{O}(M^2/E^2)$. However, this need not be the case in a non-renormalizable effective theory. It is quite admissible that in an effective theory the amplitude grows as E^2/v^2, E^4/v^4, etc. In this case the additional terms in $A(\tilde{W}\omega\omega\omega)$ do not vanish when $M^2/E^2 \to 0$. The non-unitary growth with the energy cannot continue indefinitely, of course. At some point the expansion in powers of E^2/v^2 simply breaks down and beyond that point the additional terms $A(\tilde{W}\omega\omega\omega)$ will eventually tend to zero as $M^2/E^2 \to 0$. The large M_H limit in the Standard Model amplitudes is a good example to illustrate the previous point. If M_H is very large, for $E \ll M_H$ the tree level amplitudes grow (for a while) as E^2/v^2 and the additional terms are non-negligible even for reasonably large energies.

4. Applications of the Equivalence Theorem in the Standard Model.

In this section we will explicitly check the validity of the Equivalence Theorem at tree level in the Standard Model including the first subleading corrections in (3.18), which have not been computed before. Our results clarify some misunderstandings. As we just discussed, although it is frequently stated that the next to leading terms $A(\tilde{W}\omega\omega\omega)$ are of $\mathcal{O}(M/E)$ [2,11-12] this is not always true. In particular for the scattering $W_LW_L \to W_LW_L$ when $M_H > s$ they give sizeable contributions at small angles. (In [4] the need to consider the subleading amplitudes is pointed out, but the analysis presented there only scratches the surface of the problem.) Another point we would like to stress is that the Equivalence Theorem, as stated in (3.17), is an exact result with no need of taking any limit whatsoever [11,24]. Finally, with regards to some difficulties with Lorentz invariance and the fact that one is dealing with longitudinally polarized particles which have been recently reported in [12], they do not show up if one always works in a kinematical region where *all* energies are much greater than M. All calculations have been carried out in t'Hooft-Feynman gauge. The processes $A(W^+W^- \to W^+W^-)$ and $A(W^+W^+ \to W^+W^+)$ have been calculated previously in [27] and [28], respectively.

4.1. $W^+W^- \to W^+W^-$

The exact amplitude for the scattering of W_L is given by the set of diagrams in Fig. 1

$$\begin{aligned} A(W_L^+W_L^- \to W_L^+W_L^-) =& \frac{g^2}{4M^4x^2}\Bigg[(4M^2+x)(4M^2(t^2+tx)+t^2x+4tx^2+x^3) \\ &+ M^2\frac{\left(2M^2(2t+x)+tx\right)^2}{M_H^2-t} + M^2x^2\frac{\left(2M^2+x\right)^2}{M_H^2-s} \\ &+ \left(\frac{s_w^2}{t}+\frac{c_w^2}{t-M_Z^2}\right)\Big(4M^4(x-2t)(2t^2-tx-2x^2) \\ &- 16M^6(2t+x)^2 - 8M^2(t^3x-tx^3) - t^2x^2(t+2x)\Big) \\ &- \left(\frac{s_w^2}{s}+\frac{c_w^2}{s-M_Z^2}\right)x^2(2t+x)\left(x+6M^2\right)^2\Bigg]. \end{aligned} \tag{4.1.1}$$

s, t, u are the Mandelstam variables defined by $s = (p_1+p_2)^2$, $t = (p_1-k_1)^2$ and $u = (p_1-k_2)^2$. We have introduced the variable $x = s-4M^2$ to simplify the expressions. In the CM frame

$$\begin{aligned} p_1 &= (E,0,0,p) \qquad & k_1 &= (E,p\sin\theta,0,p\cos\theta) \\ p_2 &= (E,0,0,-p) \qquad & k_2 &= (E,-p\sin\theta,0,-p\cos\theta) \end{aligned} \tag{4.1.2}$$

and

$$\begin{aligned} \epsilon_L(p_1) &= \frac{1}{M}(p,0,0,E) \qquad & \epsilon_L(k_1) &= \frac{1}{M}(p,E\sin\theta,0,E\cos\theta) \\ \epsilon_L(p_2) &= \frac{1}{M}(p,0,0,-E) \qquad & \epsilon_L(k_2) &= \frac{1}{M}(p,-E\sin\theta,0,-E\cos\theta) \end{aligned} \tag{4.1.3}$$

The following kinematical relations hold: $p^2 = x/4$ and $\cos\theta = 1 + 2t/x$.

The Goldstone boson amplitude is given by the diagrams of Fig. 2. The calculation is simpler (this is one of the main assets of the Equivalence Theorem, of course). We set $\mathcal{C} = 1$ for the time being

$$\begin{aligned} A(\omega^+\omega^- \to \omega^+\omega^-) =& g^2 \left[\frac{M_H^2}{8M^2} \frac{(M_H^2(s+t) - 2st)}{(M_H^2 - s)(M_H^2 - t)} \right. \\ & \left. +(4M^2 - 2t - s) \left(\frac{(c_w^2 - s_w^2)^2}{4c_w^2} \frac{1}{s - M_Z^2} + \frac{s_w^2}{s} \right) \right] + (s \leftrightarrow t) \end{aligned} \tag{4.1.4}$$

The first correction to the 'naive' Equivalence Theorem corresponds to the diagrams in Fig. 3. Their contribution to the r.h.s. of (3.18) is

$$\begin{aligned} A(\tilde{W}\omega\omega\omega) =& g^2 \left[\frac{M_H^2}{M^2} \left(\frac{x(M^2 - t) + t\sqrt{xs}}{x(M_H^2 - t)} + \frac{(M^2 - s) + \sqrt{xs}}{M_H^2 - s} \right) \right. \\ & \left. +4s_w^2 \left(\frac{(x^2 + x(s+t) - \sqrt{xs}(2x+t))}{2xt} + \frac{(x^2 + 2tx - \sqrt{xs}(x+2t)}{2xs} \right) \right] \\ & + g'^2 \frac{c_w^2 - s_w^2}{x} \left[\frac{(x+2t)(\sqrt{xs} - x)}{s - M_Z^2} + \frac{(2x+t)\sqrt{xs} - x(s+t+x)}{t - M_Z^2} \right] \end{aligned} \tag{4.1.5}$$

In order to check analytically the validity of the Equivalence Theorem let us expand (4.1.1) to (4.1.5) in inverse powers of the energy. Except for $\cos\theta \sim 1$, $s \to \infty \Rightarrow -t \to \infty$. Therefore we perform a double expansion in M^2/s and M^2/t

$$\begin{aligned} A(W_L^+W_L^- \to W_L^+W_L^-) =& -\frac{g^2}{4} \frac{M_H^2}{M^2} \left[\frac{t}{t - M_H^2} + \frac{s}{s - M_H^2} \right] - \frac{g^2}{2c_w^2} \frac{s^2 + t^2 + st}{st} \\ & + g^2 \frac{M_H^2}{s} \frac{2M_H^2 t - s(s+t)}{(M_H^2 - s)(M_H^2 - t)} + \mathcal{O}(M^2/s, M^2/t), \end{aligned} \tag{4.1.6}$$

$$\begin{aligned} A(\omega^+\omega^- \to \omega^+\omega^-) =& -\frac{g^2}{4} \frac{M_H^2}{M^2} \left[\frac{t}{t - M_H^2} + \frac{s}{s - M_H^2} \right] - \frac{g^2}{2c_w^2} \frac{s^2 + t^2 + st}{st} \\ & + \mathcal{O}(M^2/s, M^2/t), \end{aligned} \tag{4.1.7}$$

and, finally, (4.1.5) becomes

$$A(\tilde{W}\omega\omega\omega) = g^2 \frac{M_H^2}{s} \frac{2M_H^2 t - s(s+t)}{(M_H^2 - s)(M_H^2 - t)} + \mathcal{O}(M^2/s, M^2/t). \tag{4.1.8}$$

We have kept the complete Higgs structure in the denominator. Adding (4.1.8) and (4.1.7) reproduces (4.1.6). Notice that in the large M_H limit the additional correction to the 'naive' Equivalence Theorem is of $\mathcal{O}(1)$ in the $1/E$ expansion and this in spite of the explicit v^μ suppression factor. Indeed, although this part of the amplitude is suppressed with respect to the leading term (4.1.6) by one power of M^2/E^2, the amplitudes grow as E^2/v^2 in the large Higgs mass limit — a hint of the perturbative problems with unitarity in the

Standard Model. The additional piece $A(\tilde{W}\omega\omega\omega)$ is $\mathcal{O}(g^2)$ and thus definitely subleading w.r.t. (4.1.7) in accordance with our expectations, but not negligible in any case.

In the opposite extreme, if the Higgs is light, (4.1.5) is $\mathcal{O}(g^2 M_H^2/E^2)$, again subleading with respect to (4.1.7) , which is $\mathcal{O}(\lambda)$. But now $A(\tilde{W}\omega\omega\omega)$ does indeed vanish as $E\to\infty$, in accordance with the 'naive' statement of the Equivalence Theorem (1.1). In either case we have not included the factor $\mathcal{C}$. This would be required if we desire to work with a $\mathcal{O}(\lambda g^2)$ or $\mathcal{O}(g^2E^2/v^2)$ accuracy, but that would also require computing loop corrections.

4.2. $W^+W^+\to W^+W^+$

The diagrams entering this amplitude are similar to those of Fig. 1, but exchanging the s and u channels. The tree level results are

$$\begin{aligned}A(W_L^+W_L^+\to W_L^+W_L^+) =&\frac{g^2}{4M^4(t+u)^2}\Bigg[16M^4tu-2M^2(t+u)(t^2+6tu+u^2)\\&+\frac{1}{2}(t+u)^2(t^2+4tu+u^2)-M^2\frac{(2M^2(t-u)+tu+u^2)^2}{u-M_H^2}\\&+\left(\frac{c_w^2}{u-M_Z^2}+\frac{s_w^2}{u}\right)\big(16M^6(t-u)^2+8M^2tu(t+u)(t+2u)\\&-4M^4(t+3u)(2t^2+3tu-u^2)-u^2(t+u)^2(2t+u)\big)\Bigg]\\&+(t\leftrightarrow u),\end{aligned}\tag{4.2.1}$$

$$\begin{aligned}A(\omega^+\omega^+\to\omega^+\omega^+) =&g^2\left[\frac{M_H^2}{8M^2}\frac{(M_H^2(t+u)-2tu)}{(M_H^2-t)(M_H^2-u)}\right.\\&\left.+(4M^2-2t-u)\left(\frac{(c_w^2-s_w^2)^2}{4c_w^2}\frac{1}{u-M_Z^2}+\frac{s_w^2}{u}\right)\right]+(t\leftrightarrow u),\end{aligned}\tag{4.2.2}$$

and, for the leading correction to the 'naive' Equivalence Theorem,

$$\begin{aligned}A(\tilde{W}\omega\omega\omega) =&g^2\Bigg[\frac{M_H^2}{M^2}\frac{\left(M^2(t+u)-tu-u^2-u\sqrt{sx}\right)}{(M_H^2-u)(t+u)}+4\Bigg(\frac{s_w^2}{2u(t+u)}\\&+\frac{s_w^2(c_w^2-s_w^2)}{4c_w^2(M_Z^2-u)(t+u)}\Bigg)\Bigg(-4M^2(t+u)+2t^2+u^2+3tu\\&+(2t+u)\sqrt{sx}\Bigg)\Bigg]+(t\leftrightarrow u)\,.\end{aligned}\tag{4.2.3}$$

As in the previous case, in order to ease the comparison between these amplitudes we shall expand them up to $\mathcal{O}(M^2/E^2)$. For simplicity we work away from the forward and backward regions. Then both t and u are large and

$$\begin{aligned}A(W_L^+W_L^+\to W_L^+W_L^+) =&-\frac{g^2}{4}\frac{M_H^2}{M^2}\left[\frac{t}{t-M_H^2}+\frac{u}{u-M_H^2}\right]-\frac{g^2}{2c_w^2}\frac{t^2+u^2+tu}{tu}\\&-g^2\frac{M_H^2}{(t+u)}\frac{(t-u)^2}{(M_H^2-t)(M_H^2-u)}+\mathcal{O}(M^2/t,M^2/u),\end{aligned}\tag{4.2.4}$$

$$A(\omega^+\omega^+ \to \omega^+\omega^+) = -\frac{g^2}{4}\frac{M_H^2}{M^2}\left[\frac{t}{t-M_H^2} + \frac{u}{u-M_H^2}\right] - \frac{g^2}{2c_w^2}\frac{t^2+u^2+tu}{tu} + \mathcal{O}(M^2/t, M^2/u), \tag{4.2.5}$$

and for the additional piece in this limit we get

$$A(\tilde{W}\omega\omega\omega) = -g^2\frac{M_H^2}{(t+u)}\frac{(t-u)^2}{(M_H^2-t)(M_H^2-u)} + \mathcal{O}(M^2/t, M^2/u). \tag{4.2.6}$$

The addition of eq.(4.2.5) and (4.2.6) reproduces eq. (4.2.4). Now, however, (4.2.6) is $\mathcal{O}(g^2E^2/M_H^2)$ for large values of the Higgs mass. Since the leading term is still $\mathcal{O}(E^2/v^2)$, the leading corrections to the 'naive' Equivalence Theorem are down by a factor M^2/M_H^2 which is actually smaller than M^2/E^2 in this limit. For a light Higgs, the correction is $\mathcal{O}(g^2M_H^2/E^2)$, to be compared with the leading $\mathcal{O}(\lambda)$ contribution, this time in accordance with the usual counting.

Of course, both in this case and in the previous one, the additional contributions can be greatly enhanced by some kinematical reasons, e.g. in the vicinity of the Higgs pole. We discuss this issue in more detail in the next subsection.

4.3. Domain of Applicability

Let us now analyze more carefully the improvement on the Equivalence Theorem that is brought about by keeping the additional terms in (3.18), such as (4.1.5) and (4.2.3).

We will discuss here the scattering $W_L^+W_L^- \to W_L^+W_L^-$ whose tree level results have been described in section 4.1. We have plotted this amplitude for three different angles ($\theta = \pi/16, \pi/4, 3\pi/4$) (Fig. 4). We take as physical input M_Z, M, M_H and α and work in the region $2M \ll E \ll M_H$. We have taken $M_H = 1$ TeV.

The solid line corresponds to the exact tree level result for W_L scattering. The short-dashed line corresponds to the $g = g' = 0$ limit, which is the standard approximation in the literature (see [2,28] and the second reference in [27]). Notice that the corrections to the 'naive' Equivalence Theorem are always proportional to g^2. From Fig.4 it is clear that setting $g = 0$ is a very crude approximation, particularly at small angles, and that the difference does not go to zero as $E \to \infty$ since the additional terms that correct the 'naive' Equivalence Theorem are not of $\mathcal{O}(M^2/E^2)$ but rather *down* by a factor M^2/E^2 with respect to the leading contribution, which is quite different. In fact their effect may be quite sizeable.

The corrections for $g \neq 0$ have two origins. On the one hand the first term on the r.h.s. of (3.18), which corresponds to the 'naive' Equivalence Theorem, gets contributions from the exchange of γ and Z. Adding these corrections (dashed-dotted line) improves the agreement with the exact result substantially but still fails to reproduce the scattering amplitude of longitudinal W's in many kinematical regions. When we finally add the correction contained in (4.1.5) the result (represented by a long dashed line) is practically indistinguishable from the exact one for all kinematical regions. In fact it is so close as to become invisible for most angles. The terms proportional to $(v^\mu)^2$ and beyond in (3.18) are obviously unimportant.

At this stage one should stress that it is totally unnecessary to expand the amplitudes in inverse powers of M_H to verify the consistency of the Equivalence Theorem as is sometimes done [24]. The full analytical structure of the Higgs propagator is well reproduced (including $\mathcal{O}(M^2/E^2)$ corrections) by adding the terms proportional to v^μ. It is also illustrative to consider a complete $1/E$ expansion of the Z propagators contained in the amplitudes. The results (represented by a dotted line in the Fig. 4. (b)) are obviously much worse.

5. Effective Chiral Lagrangian

So far we have applied the Equivalence Theorem at tree level in the minimal Standard Model. It is far more interesting to go beyond this level and apply the above results at higher orders in the perturbative expansion or, better, to use the connection that it provides between scattering of longitudinal W's and Goldstone bosons to set bounds on new physics in the longitudinal sector. The level of precision required in the latter case is typically also that of a radiative correction since tree level type modifications are by now excluded.

It is convenient and economical to treat the minimal Standard Model and other theoretical possibilities on the same footing[29]. Provided that the Higgs mass is sufficiently large, this can be achieved by working with an effective chiral lagrangian. This consists of a collection of operators with the required symmetry properties of $SU(2)_L \times U(1)_Y$ local gauge invariance and containing the Goldstone bosons of the $SU(2)_L \times SU(2)_R \to SU(2)_V$ global symmetry breaking. These two conditions greatly restrict the possible operators. Chiral and gauge invariance force the interactions to be derivative and effective operators can be classified according to the powers of momenta.

The most general Electroweak Chiral Lagrangian up to $\mathcal{O}(p^4)$ is of the form

$$\mathcal{L}^{eff} = -\frac{1}{2}\mathrm{Tr}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{v^2}{4}\mathrm{Tr}D_\mu U^\dagger D^\mu U + \sum_{i=0,13} a_i\mathcal{L}_i + \mathcal{L}_{GF} + \mathcal{L}_{FP} \quad (5.1)$$

The complete list of operators is given in Appendix C. The $SU(2)_L \times U(1)_Y$ gauge symmetry is realized non-linearly at the level of the Goldstone bosons (see (2.2.3) and (2.2.4)).

For a sufficiently large Higgs mass, the minimal Standard Model is just a particular case of (5.1). The actual value of the set of coefficients $\{a_i\}$ that correspond to the minimal Standard Model is obtained from (2.2.5), (2.2.6) and (2.2.7) after integrating out the ρ field. The safest way to obtain their value is through the matching conditions[7,18-19,30], in which one requires equivalent descriptions in terms of fundamental and effective theories, therefore determining the values of $\{a_i\}$. The matching is done in perturbation theory at the one loop level. Although it would be clearly desirable to go beyond this, no results are available at present.

The matching of both theories requires some care[19] due to the subtleties of gauge invariance and gauge-fixing. The matching could *a priori* be carried out at different levels: S-matrix elements, connected Green functions, effective action, etc. The softest requirement is to demand equal physical S-matrix elements. This method is bound to work in all cases[21], but, given the large number of possible operators in the effective theory, it is

cumbersome. It is important to remember that whenever one takes advantage of an effective theory to describe a physical system one is using a different (sometimes coarser) set of variables to describe the Hilbert space of the system. There is absolutely no guarantee that anything other than observables should agree when using two different sets of variables. In fact, it may not even be possible to pose the question meaningfully. Therefore, the use of Green functions to perform the matching is, generally speaking, ruled out. Fortunately, for the case at hand it was shown in [19] that it is possible (and actually simplest) to match the fundamental and effective theories for renormalized connected Green functions containing only gauge fields. This is because the gauge fields are insensitive to the way the scalar sector is parametrized. On the other hand, requiring the matching at the level of the effective action (generating functional of 1PI diagrams) as it is sometimes done is not consistent.

When matching renormalized connected Green functions between the fundamental and the effective theory one must be careful to project out the longitudinal parts. These are gauge dependent and there is no guarantee that the gauge in the fundamental and in the effective theory are the same, since by definition this is not observable. Thus it is not guaranteed that with the set of gauge invariant operators contained in (5.1) one should be able to reproduce the longitudinal parts of the Green functions. Rather one will in general need to consider also other BRS invariant (but not gauge invariant) operators of the right dimensionality to proceed with the matching. In practice this means that there are some coefficients in the lagrangian (5.1) which cannot be determined.

In the Effective Chiral Lagrangian there are operators that either vanish or simply reduce to other operators when the equations of motion are used. They correspond to the coefficients a_{11}, a_{12} and a_{13}. Therefore it is quite clear that working at the one loop level they will never be determined via S-matrix elements. It turns out that they cannot be determined via renormalized Green functions either because they contribute to the gauge longitudinal parts which, as discussed, require additional BRS invariant operators to match and we end up with more unknowns than matching equations. (a_{11}, ... contribute to other Green functions as well, but these involve Goldstone bosons, which are also unphysical) Then, the longitudinal part of Green functions cannot be unambiguosly fixed in an effective theory. Yet the Equivalence Theorem is basically concerned with longitudinal parts. Is the Equivalence Theorem in jeopardy in an Effective Chiral Lagrangian? We will return to this crucial point in the next section.

If one is interested in reproducing the minimal Standard Model at tree level for energies $E \ll M_H$ it is enough to keep a_5 and set

$$a_5^{tree} = \frac{v^2}{8M_H^2}. \tag{5.2}$$

At the one loop level one requires the full expression for a_5 and the other coefficients, which can be found in [7,18-19,30]. The natural expansion parameter in an Effective Chiral Lagrangian being E^2/v^2 (or rather $E^2/(4\pi v)^2$), the effective theory lends itself very easily to the sort of energy expansion that is a characteristic of the Equivalence Theorem.

The amplitude for the scattering of longitudinally polarized W's takes the symbolic

form

$$
\begin{aligned}
A =&(b_1^{(0)} g^2 + b_2^{(0)} g^2 \frac{M^2}{E^2} + \ldots)(1 + \mathcal{O}(\frac{g^2}{16\pi^2}))\\
&+ \frac{E^2}{v^2}(b_1^{(2)} + b_2^{(2)} \frac{g^2}{16\pi^2} + \ldots)\\
&+ \frac{E^4}{16\pi^2 v^4}(b_1^{(4)} + b_2^{(4)} \frac{g^2}{16\pi^2} + \ldots)\\
&+ \ldots
\end{aligned}
\tag{5.3}
$$

The first line on the r.h.s. of (5.3) has its origin in tree level exchange of vector bosons, once expanded in powers of E. The interesting physics is in the a_i coefficients that are contained in the constants $b_1^{(4)}$ and $b_2^{(2)}$. Clearly, to make any definite statements on these coefficients via the Equivalence Theorem we need to be able to compute the r.h.s. of (3.18) (or rather its counterpart in an Effective Chiral Lagrangian) with enough accuracy. In previous sections we have seen that the 'naive' Equivalence Theorem has corrections that modify the leading term by factors of $\mathcal{O}(M^2/E^2)$. If this also holds in an effective theory, we need to assume that M^2/E^2 is small, for the Equivalence Theorem to be of practical use. On the other hand, it must be satisfied that $E^2 \ll 16\pi^2 v^2 = 64\pi^2 M^2/g^2$. This seems to provide a reasonably large window of applicability. Of course this window must get bigger when we include more and more corrections on the r.h.s. of (3.18). It is our contention that adding the first non-leading corrections is enough for practical applications.

If we keep in our effective lagrangian terms of $\mathcal{O}(p^4)$ at most and work at the one loop order only terms of up to $\mathcal{O}(E^4/v^4)$ will be generated in the different amplitudes appearing in (3.18). Since there is a suppression factor M^2/E^2 due to the v^μ factor, the correction to the 'naive' Equivalence Theorem will produce terms of $\mathcal{O}(g^2)$ and $\mathcal{O}(g^2 E^2/v^2)$ as well as terms that are suppressed by powers of M^2/E^2. Further corrections (terms with two v^μ or more) would produce contributions either of $\mathcal{O}(g^4)$ or right away suppressed by powers of M^2/E^2. Clearly at large energies (but still much less than $4\pi v$), the relevant contributions will be contained in the two terms that we keep on the r.h.s. of (3.18). Since factors of 4π, etc may be relevant, our claim can only be fully justified by a detailed calculation which is presented in section 7.

6. The Equivalence Theorem and the Effective Chiral Lagrangian

In the effective theory we shall use the same gauge condition as in the non-linear realization of the minimal Standard Model

$$
F^{\pm} = \frac{1}{\sqrt{\xi_1}} \partial^\mu W_\mu^{\pm} - M\sqrt{\xi_2}\pi^{\pm}. \tag{6.1}
$$

Since the derivation of the Equivalence Theorem hinges on the use of Ward identities, it is not difficult to see that all steps hold the case of an Electroweak Chiral Lagrangian.

Therefore

$$
\begin{aligned}
A(W_L^+W_L^- \to W_L^+W_L^-) =& \mathcal{C}^4 A(\pi^+\pi^- \to \pi^+\pi^-) \\
&+ i\mathcal{C}^3 A(\tilde{W}^+\pi^- \to \pi^+\pi^-) + i\mathcal{C}^3 A(\pi^+\tilde{W}^- \to \pi^+\pi^-) \\
&- i\mathcal{C}^3 A(\pi^+\pi^- \to \tilde{W}^+\pi^-) - i\mathcal{C}^3 A(\pi^+\pi^- \to \pi^+\tilde{W}^-) \\
&+ \mathcal{O}((v^\mu)^2).
\end{aligned}
\tag{6.2}
$$

In addition, in obtaining the formal expression for the $\mathcal{C}$ factor nothing depends on the particular theory we are using; it is just a consequence of the Ward identities of the theory. Being completely general, eq. (3.14) carries over to the Effective Chiral Lagrangian merely replacing Σ_ω and Z_ω by Σ_π and Z_π. Because $\mathcal{C}$ is finite when $M_H \to \infty$ in the minimal Standard Model, it stays finite in an Electroweak Chiral Lagrangian[6].

It is perhaps useful to start our discussion by choosing the values of the a_i coefficients [7,18-19,30] that reproduce the minimal Standard Model. Let us emphasize that 'reproducing the Standard Model' does not imply that bare self-energies and renormalization constants have to be numerically equal to those used in section 4. In general they will not be. But physical amplitudes will. The bare self-energies will have two types of contributions: from the $\mathcal{O}(p^2)$ lagrangian (including in this the gauge part), entering both at tree level and at the one loop level, and from the $\mathcal{O}(p^4)$ lagrangian, entering only at tree level, according to the usual chiral counting rules. The $\mathcal{O}(p^4)$ contribution to the self-energies is given in Appendix C.

It is quite instructive to repeat the verification of the Equivalence Theorem at tree level in the minimal Standard Model in the language of the Effective Chiral Lagrangian. The amplitudes $A(W^+W^- \to W^+W^-)$ or $A(W^+W^+ \to W^+W^+)$ come out exactly as in (4.1.1) and (4.2.1), except that they appear expanded in inverse powers of M_H. The left hand side of (3.18) changes completely. There is a reshuffling of different contributions between the leading and the subleading terms. In particular, one can easily see that the amplitudes $A(\pi^+\pi^- \to \pi^+\pi^-)$ and $A(\pi^+\pi^+ \to \pi^+\pi^+)$ have changed. This should be no surprise as they are not physical amplitudes and may perfectly be different in the Standard Model and in its Effective Chiral Lagrangian (compare formulae (4.1.7) and (7.1.7)).

At the one loop level we have to use the values derived in [7,18-19,30] for the $\{a_i\}$, or simply keep them arbitrary if we wish to parametrize different alternatives to the minimal Standard Model. At this order we will face the problem of the uncertainties in the longitudinal components of the Green functions we have alluded to before. To be definite we will pick a particular operator that contributes to the longitudinal parts, such as the one with coefficient a_{11}, and follow its track through the different contributions in (6.2). a_{11} might, on dimensional grounds, appear in principle as a contribution of $\mathcal{O}(E^4/v^4)$ through the diagram with four π's. However, the structure of the operator $\mathcal{L}_{11}$ is such that the contribution is $\mathcal{O}(g^2E^2/v^2)$. In addition a_{11} may show up as a contribution via radiative corrections to any of the two amplitudes on the r.h.s. of (6.2) or via $\mathcal{C}$. The contribution would be in either case of $\mathcal{O}(g^2E^2/v^2)$. In conclusion, although a_{11} appears almost everywhere in the course of the calculation, at the end of the day a_{11} should drop from the r.h.s. of the Equivalence Theorem because a_{11} cannot appear on the l.h.s. given that $A(W_LW_L \to W_LW_L)$ is physical and we are working at the one loop level (a_{11} could conceiveably appear at the two loop order). Let us see this in detail.

The diagrams to compute are depicted in Fig.5. We shall work in Landau gauge, but we have checked the cancellation of the gauge dependence. The pion amplitude has two types of contributions proportional to a_{11}. On the one hand, the diagram (a) of Fig. 5 gives

$$-\frac{4g^2}{v^2}(s+t)a_{11}. \tag{6.3}$$

(Only the part of the amplitude proportional to a_{11} is presented here.) On the other hand there is a contribution to the external legs represented by (b). There are four such diagrams. Adding the four of them one gets

$$\frac{4g^2}{v^2}(s+t)a_{11}. \tag{6.4}$$

Diagram (c) vanishes in Landau gauge. The total contribution from diagrams with external Goldstone bosons vanishes. Finally, the amplitude with one $W^{\pm}$ and three Goldstone bosons gets contributions from diagrams (d) and (e), which respectively give

$$-\frac{4g^2}{v^2}(s+t)a_{11} \qquad \frac{4g^2}{v^2}(s+t)a_{11}. \tag{6.5}$$

The bare amplitudes do not depend on a_{11}. The renormalization constants and self-energies (Appendices A and C) entering $\mathcal{C}$ do depend on a_{11}, however. Therefore both $\mathcal{C}$ and the renormalized amplitudes are potentially dependent on a_{11}. Yet, in Landau gauge, which we are using, $\mathcal{C}$ turns out to be a_{11}-free and so are the renormalized amplitudes. The moral is that the Green functions that appear in the formulation of the Equivalence Theorem are one by one potentially ambiguous but the ambiguities drop in physical quantities. In practice there is no need to go through the painstaking process of constructing BRS invariant operators, matching them and keeping track of these spureous longitudinal parts.

7. Applying the Equivalence Theorem to the Effective Chiral Lagrangian

As discussed in section 5, one is working here within an energy expansion. On the other hand, the Equivalence Theorem implies an expansion in inverse powers of the energy. It is obvious that these two expansions can give at best a window of applicability. The question whether this window is of zero or negligible width has been recently raised in [4,13]. These authors have considered the $g=0$ approximation. Numerical analysis[13] show that then the Equivalence Theorem holds only for very high energies, sometimes higher than the regions where chiral perturbation theory can be trusted. We would like now to substantiate the claim that keeping the first leading corrections to the 'naive' Equivalence Theorem is enough to restore the agreement.

7.1. $A(W^+W^- \to W^+W^-)$

Here we will be interested in analyzing this amplitude, already studied in the minimal Standard Model at tree level, from the point of an Effective Chiral Lagrangian. We will consider the lowest order contribution $\mathcal{O}(g^2)$, $\mathcal{O}(p^2)$ plus the contribution from higher

dimensional operators $\mathcal{O}(g^4)$, $\mathcal{O}(g^2p^2)$ and $\mathcal{O}(p^4)$ that will explicitly depend on the $\{a_i\}$ coefficients. We shall *not* include the one loop $\mathcal{O}(g^4)$, $\mathcal{O}(g^2p^2)$ and $\mathcal{O}(p^4)$ contributions that, at the same order, should be taken into account. This is of course *not* quite correct, but it allows us to give short closed expressions. Furthermore, this is enough to trace the $\log M_H$ dependence in the Standard Model, or the dependence in the new physics in other models, and argue the different pros and cons of the several approximations that can be made when dealing with the Equivalence Theorem.

The exact amplitude for the scattering of four W_L's is obtained from Fig. 6. The result is

$$\begin{aligned}
A(W_L^+W_L^- \to W_L^+W_L^-) =& \frac{1}{4M^4x^2}\Big\{C_1(4M^2+x)\Big[4M^2(t^2+tx)+x(t^2+x^2+4tx)\Big] \\
&+ C_2\Big[8M^4(2t^2+x^2+2tx)+4M^2x(2t^2+x^2+tx) \\
&+ x^2(t^2+x^2)\Big] + S_1\Big[-x^2(6M^2+x)^2(2t+x)\Big] \\
&+ S_2\Big[-16M^6(2t+x)^2+4M^4(x-2t)(2(t^2-x^2)-tx) \\
&+ 8M^2(tx^3-xt^3)-t^2x^2(t+2x)\Big] \\
&+ \sum_{V=\gamma,Z}\frac{1}{M_V^2-t}\Big[A_V^2\Big(-16M^6(2t+x)^2-8M^2(t^3x-x^3t) \\
&+ 4M^4(x-2t)(2(t^2-x^2)-tx)-t^2x^2(t+2x)\Big) \\
&+ 2A_VB_Vt(4M^2+x)\left(2M^2(2(t^2-x^2)-tx)+tx(t+2x)\right)\Big] \\
&+ \sum_{V=\gamma,Z}\frac{1}{M_V^2-s}\Big[A_V^2\left(-x^2(6M^2+x)^2(2t+x)\right) \\
&+ 2A_VB_Vx^2(4M^2+x)(6M^2+x)(2t+x)\Big]\Big\},
\end{aligned} \tag{7.1.1}$$

where C_1 and C_2 are defined by

$$\begin{aligned}
C_1 &= g^2\left\{1+g^2(a_4+a_8)-2g^2(a_3+a_9-a_{13})\right\}, \\
C_2 &= 2g^4(a_4+a_5).
\end{aligned} \tag{7.1.2}$$

The contribution from the self energies of the exchanged vector bosons is included in the quantities S_1 and S_2 in (7.1.1)

$$S_1 = -g^2\left(c_w^2\frac{1}{(s-M_Z^2)^2}\Sigma_Z(s)+s_w^2\frac{1}{s^2}\Sigma_\gamma(s)+2s_wc_w\frac{1}{s-M_Z^2}\frac{1}{s}\Sigma_{\gamma Z}(s)\right),$$

$$S_2 = -g^2 \left(c_w^2 \frac{1}{\left(t - M_Z^2\right)^2} \Sigma_Z(t) + s_w^2 \frac{1}{t^2} \Sigma_\gamma(t) + 2 s_w c_w \frac{1}{t - M_Z^2} \frac{1}{t} \Sigma_{\gamma Z}(t) \right). \qquad (7.1.3)$$

The contribution from a_i to the self-energies involved is given in Appendix C. Finally M_V stands for the vector boson mass ($V = \gamma, Z$) and

$$\begin{aligned} A_\gamma &= -igs_w \qquad A_Z = -igc_w \left(1 - \frac{1}{c_w^2} a_3 g^2\right) \\ B_\gamma &= -ig^3 \left(s_w a_3 + s_w(a_1 - a_2) - s_w(a_8 - a_9)\right) \\ B_Z &= -ig^3 \left(-\frac{s_w^2}{c_w} a_3 + \frac{s_w^2}{c_w}(a_2 - a_1) - c_w(a_8 - a_9) - \frac{1}{c_w} a_{13}\right) \end{aligned} \qquad (7.1.4)$$

Expanding the exact amplitude (7.1.1) in inverse powers of v^2 one gets

$$\begin{aligned} A(W_L^+ W_L^- \to W_L^+ W_L^-) =& \frac{4}{v^4} \left[(3(s^2 + t^2) + 4st)(a_4 + a_{13}) + 2(s^2 + t^2)(a_5 - a_{13}) \right] \\ & + \frac{(s+t)}{v^2} \Big[1 + 6a_0 + 6g'^2 a_2 - 2g^2(a_3 - a_9) \\ & - g^2 \frac{(s - 2t)}{s} \left(12(a_4 + a_{13}) + 8(a_5 - a_{13})\right) \Big] \\ & - \frac{g^2}{2c_w^2} \frac{1}{st} (s^2 + t^2 + st - 4t^2 c_w^2) + \mathcal{O}(g^4). \end{aligned} \qquad (7.1.5)$$

This result has to be compared with the corresponding scalar amplitude (Fig. 7 (a)-(e)). The Goldstone boson amplitude at tree level is

$$\begin{aligned} A(\pi^+\pi^- \to \pi^+\pi^-) =& \frac{4}{v^4} \left[(3(s^2 + t^2) + 4st)(a_4 + a_{13}) + 2(s^2 + t^2)(a_5 - a_{13}) \right] \\ & + \frac{1}{v^2}(s+t) \Big[1 + 6a_0 - g^2(12(a_4 + a_{13}) + 8(a_5 - a_{13})) \\ & + 12 g^2 s_w^2 (a_2 - a_3 - a_9) \Big] - \frac{2}{3} g^2 - 2g^2 s_w^2 \frac{(s^2 + t^2 + st)}{st} \\ & - 8g^4 s_w^2 (a_2 - a_3 - a_9) + 4g^4(a_4 + a_5) + g^4 v^2 s_w^2 \frac{(s+t)}{st} \\ & + 2 \frac{g^2}{v^2} \frac{(c_w^2 - s_w^2)}{c_w^2} \frac{(s + 2t - 4M^2)}{M_Z^2 - s} \Big(c_w^2 (a_3 + a_9) s + s_w^2 a_2 s \\ & + \frac{1}{2} v^2 a_0 + \frac{v^2}{8}(c_w^2 - s_w^2) \Big) \\ & + 2 \frac{g^2}{v^2} \frac{(c_w^2 - s_w^2)}{c_w^2} \frac{(t + 2s - 4M^2)}{M_Z^2 - t} \Big(c_w^2 (a_3 + a_9) t + s_w^2 a_2 t \\ & + \frac{1}{2} v^2 a_0 + \frac{v^2}{8}(c_w^2 - s_w^2) \Big) + self - energies \end{aligned} \qquad (7.1.6)$$

where '*self-energies*' stands for diagrams with self-energy insertions in the gauge propagators in (a),(b),(d) and (e). They are of $\mathcal{O}(g^4)$. Expanding as before

$$
\begin{aligned}
A(\pi^+\pi^- \to \pi^+\pi^-) =& \frac{4}{v^4}\Big[(3(s^2+t^2)+4st)(a_4+a_{13})+2(s^2+t^2)(a_5-a_{13})\Big] \\
&+\frac{1}{v^2}(s+t)\Big[1+6a_0+6g'^2a_2-6g^2(a_3+a_9) \\
&-g^2\left(12(a_4+a_{13})+8(a_5-a_{13})\right)\Big] \\
&-\frac{g^2}{2c_w^2}\frac{1}{st}(s^2+t^2+st+\frac{4}{3}stc_w^2)+\mathcal{O}(g^4).
\end{aligned}
\tag{7.1.7}
$$

Finally, the contributions with one power of v^μ lead to the diagram in Fig. 7 (f). All the other diagrams entering in this amplitude are of $\mathcal{O}(g^4)$ and need not be included.

$$
\begin{aligned}
A(\tilde{W}\pi\pi\pi) =& \frac{8}{v^4x}\left(\sqrt{sx}-s\right)\Big[(6(s^2+t^2)+8st)(a_4+a_{13})+4(s^2+t^2)(a_5-a_{13})\Big] \\
&+\frac{4}{v^2x}\left(\sqrt{sx}-s\right)(s+t)+\frac{8}{v^2x}g^2\Big[\left(6(2s^2+t^2)+14st-\sqrt{sx}(9s+7t)\right)(a_4 \\
&+a_{13})+\left(4(2s^2+t^2)+4st-2\sqrt{sx}(3s+t)\right)(a_5-a_{13})+\frac{\sqrt{sx}}{2}(s+t)(a_3 \\
&+2a_9)\Big]+\frac{4}{3x}g^2\Big(3(s+t)+2x-12g^2(2(s+t)+x)(a_5-a_{13}) \\
&-12g^2(3(s+t)+x)(a_4+a_{13})\Big)-4g^2\frac{\sqrt{sx}}{x}\Big(1+g^2(a_3-6(a_4+a_{13}) \\
&-4(a_5-a_{13})+2a_9)\Big).
\end{aligned}
\tag{7.1.8}
$$

The expanded amplitude is

$$
A(\tilde{W}\pi\pi\pi)=\frac{4g^2}{v^2}\frac{(s+t)}{s}\Big[sa_3+6t(a_4+a_{13})+4t(a_5-a_{13})+2sa_9\Big]+2g^2\left(\frac{1}{3}+\frac{t}{s}\right)+\mathcal{O}(g^4).
\tag{7.1.9}
$$

Adding up eq. (7.1.7) and (7.1.9) one recovers the result (7.1.5). This is a nice example of the verification of the Equivalence Theorem and we will use it in a moment to analyze numerically which are the most relevant pieces that one should take into account depending on the range of energies. Up to now the Equivalence Theorem has been mostly considered in the $g=g'=0$ limit (a small subset of the previous formulae) and found lacking. The additional terms take good care of the discrepancies.

It is interesting to see that one can easily determine a_5^{tree} from a comparison between the formulae derived in this section and (4.1.6) in the large M_H limit. Expanding the

latter expression in inverse powers of M_H we get

$$\begin{aligned}A(W_L^+W_L^- \to W_L^+W_L^-) =& \frac{1}{v^2}(s+t) - \frac{g^2}{2c_w^2}\frac{1}{st}(s^2+t^2+st-4t^2c_w^2) \\ &+ \frac{1}{M_H^2}\left(\frac{1}{v^2}(s^2+t^2) + \frac{g^2}{s}(-s^2+2t^2+st)\right) \\ &+ \mathcal{O}(M^2/s) + \mathcal{O}(1/M_H^4).\end{aligned} \tag{7.1.10}$$

This coincides exactly with the amplitude (7.1.5) if one substitutes $a_5 = v^2/8M_H^2$ and sets the rest of a_i's equal to zero. The same check can be done for the other amplitudes.

In processes involving only charged W's there is no dependence on the two remaining $\mathcal{O}(p^4)$ operators $\mathcal{L}_6$ and $\mathcal{L}_7$. They appear in processes such as $WW \to ZZ$ involving external Z's. The values of these coefficients in the minimal Standard Model are actually best determined by comparing the S-matrix elements for these processes in the Effective Chiral Lagrangian and in the minimal Standard Model and making use of the Equivalence Theorem itself with $g = g' = 0$. Notice that they give contributions of $\mathcal{O}(E^4/v^4)$ and thus are formally unaffected by the additional subleading corrections. (Of course, *testing experimentally* these —and the other— coefficients is another matter and there one would have to keep the subleading additional terms and the $\mathcal{O}(g^4)$, $\mathcal{O}(g^2p^2)$ contribution that we have not considered at all.)

7.2 Domain of Applicability

Finding the domain of applicability (or rather the 'domain of usefulness') of the Equivalence Theorem in the framework of the Effective Chiral Lagrangian is much more subtle than in a renormalizable theory such as the minimal Standard Model (with a light Higgs). There is a competition between two type of expansions: the natural in an effective theory in powers of the energy (over some scale), and the expansion in inverse powers of the energy (normalized by some other scale), peculiar to the Equivalence Theorem. The ratio between these two scales and the number of terms one takes in each expansion will determine the window of applicability.

By considering the process described in detail in section 7.1 we shall try to learn about the above questions. We shall put $a_5 = v^2/8M_H^2$ and $a_i = 0$ for $i \neq 5$. $\mathcal{C}$ is set equal to one. This is the choice of coefficients that corresponds to treating the minimal Standard Model at tree level for $E \ll M_H^2$. For us, however, is just a choice of $\mathcal{O}(p^4)$ coefficients; the analysis could well be repeated in the same way for any other choice. Our approach will be similar to the one taken in section 4.3. We shall consider only energies where keeping, at most, the $\mathcal{O}(p^4)$ terms in the effective action is meaningful.

We have plotted in Fig 8. the different contributions to the amplitudes the same three angles as in section 4.3 ($\theta = \pi/16, \pi/4, 3\pi/4$). The solid line corresponds to the exact $A(W_L^+W_L^- \to W_L^+W_L^-)$ amplitude (i.e. to the l.h.s. of the Equivalence Theorem). The short-dashed line is the Goldstone boson amplitude with $g = g' = 0$ (what is referred in the text as the 'naive' Equivalence Theorem.) It is clear that this approximation, particularly for small angles, is quite far from the exact result.

The full Goldstone boson amplitude $A(\pi^+\pi^- \to \pi^+\pi^-)$ with $g \neq 0$ is represented by the dash-dotted line. The improvement is quite impressive for most of the angles quoted. However some small discrepancies appear at the 10% level in the backward direction at low s (case (c)). In fact in that region setting $g \neq 0$ worsens the agreement with the exact amplitude. For large values of s the agreement between the dash-dotted line and the exact result is actually better than in the Standard Model (Fig. 4). This is because for such large values of s the Standard Model results are being very sensitive to the Higgs pole (there we took $M_H = 1$ TeV), while here in the effective theory we have obviously no such pole. The cancellation of leading and next to leading effects is thus more subtle in the Standard Model that in the effective theory when s approaches 1 TeV.

Finally we include the additional $A(\tilde{W}\pi\pi\pi)$ amplitude, the first subleading correction to the Equivalence Theorem. The $A(\tilde{W}\pi\pi\pi)$ is $\mathcal{O}(g^2)$ and the result of adding it to the dash-dotted line is represented by a long-dashed line, but the reader will not be able to see it in Fig. 8, except for very small values of s in the backward direction (c). It just overlaps very nicely with the exact result almost everywhere.

One could also expand all the contributions in powers of $1/v^2$. This is not a good idea, however. As we see from (a) the agreement is rather poor (it also was in the Standard Model) at low values of s and small angles. This is not really a difficulty of the Equivalence Theorem; if the $A(W_L^+W_L^- \to W_L^+W_L^-)$ amplitude is also expanded there is perfect agreement between the l.h.s. and the r.h.s. of the Equivalence Theorem. Only that they do not reproduce the exact results. The culprit are the diagrams with Z exchange at tree level. They are to be kept without attempting any expansions.

Up to now we have not been very concerned with the fact that we are dealing with an energy expansion cut at $\mathcal{O}(E^4)$. The first term that we are throwing away in this expansion in the effective chiral description of the Standard Model is one of $\mathcal{O}(E^6/v^2M_H^4)$ (from tree-level exchange of the Higgs and assumming that $M_H = 1$ TeV). This dictates an upper bound to the region of applicability of the Effective Theory around $E \sim 0.6$ TeV (for this value of the Higgs mass). This can be seen by comparing Figs. 4 and 8. This upper bound obviously depends on the Higgs mass; the higher the Higgs mass the larger the region of coincidence between the Standard Model and the Effective Chiral Lagrangian (with the appropriate choice of a_i coefficients, of course). In any case there is a limiting scale of applicability; since $4\pi v \simeq 3$ TeV it lies probably around 1.5 TeV.

Are the improvements brought about to the 'naive' Equivalence Theorem necessary to draw physical consequences from experiments? The answer is obviously positive. In Fig. 9 we have changed the value of the coefficient a_5 from $v^2/8M_H^2$, with $M_H = 1$ TeV to $N_{TC}/384\pi^2$, with $N_{TC} = 16$ (a popular value in some Technicolor models) and plotted the results for $\theta = \pi/5$. The figure speaks for itself.

8. Conclusions

In the previous pages we have tried to convey the idea that the Equivalence Theorem is much more than an easy way of getting order-of-magnitude estimates for amplitudes of processes involving longitudinally polarized W's and Z's. By carefully keeping track of the next-to-leading corrections it is possible to compute those amplitudes in term of other ones involving Goldstone bosons, always evaluated at $k^2 = M^2$, plus some terms involving just

one external W or Z (also evaluated at $k^2 = M^2$), with an accuracy that it is good enough to discern different types of potential 'new physics' in the symmetry breaking sector of the Standard Model. Not only are the calculations technically more convenient and easy when done with external Goldstone bosons, but also conceptually clearer, as they are more prone to comparison with other physical models such as the strong chiral lagrangian.

We have considered the minimal Standard Model written in the usual linear realization (the right framework for a light Higgs) and the Effective Electroweak Chiral lagrangian that encompasses both a heavy Higgs and other theoretical possibilities in which new physics, characterized by a scale Λ, would creep in through the $\mathcal{O}(p^4)$ effective operators. It is a must that we are sensitive to these effective operators, at least for a range of energies. Otherwise the whole approach would be useless.

We have seen that for a light Higgs some additional corrections that we have considered (those involving diagrams with all but one of the W's replaced by Goldstone bosons) can indeed be neglected at high energies (but still much lower than M_H). This result ultimately hinges on the perturbative renormalizability of the model. As soon as we get very close to M_H the additional terms start becoming more relevant, even for $E^2 \gg M^2$, as they are for exceptional momenta configurations in the forward and backward directions. We have always to keep this in mind.

Other corrections such as the multiplying factor $\mathcal{C}$ are clearly necessary if one wants to work with a one loop precision. We have for the first time provided all the necessary ingredients to go beyond tree level by determining the $\mathcal{C}$ coefficient. These corrections have been worked out in the usual on-shell scheme. This is clearly more useful for practical use than other theoretical possibilities such as non-local gauge fixing terms[8] or other schemes which are defined in somewhat vage terms[5,10].

In the context of the Effective Chiral Theory, the usual power counting arguments that have been commomly put forward when employing the Equivalence Theorem take a new twist. It should be clearly stated that the Equivalence Theorem is perfectly valid in the effective theory. As far as energy power counting arguments go, the Equivalence Theorem is both easier and more difficult in the effective theory. For one thing it is not always true that the corrections usually lumped under the line $\mathcal{O}(M^2/E^2)$ can always be neglected. Because in a non-renormalizable theory the amplitudes may grow with the energy, these corrections turn out to be relevant and are required to test the $\mathcal{O}(p^4)$ terms in the effective lagrangian. On the other hand, since the Higgs has been removed from the spectrum the kinematical singularities that lead to 'abnormal' contributions from the higher order contributions in the $1/E$ expansion are absent.

In addition to being valid, the Equivalence Theorem remains very useful in an Effective Chiral Lagrangian. It is true that one must include the $\mathcal{C}$ factor and the additional $A(\tilde{W}\pi\pi\ldots)$ piece, but $\mathcal{C}$ in the on-shell scheme depends only on one-loop self energies and it is finite in the Effective Chiral Lagrangian. Furthermore the $A(\tilde{W}\pi\pi\ldots)$ additional amplitude needs only to be computed at tree level since loop corrections would be too small. Other corrections are completely negligible. In conclusion, the Equivalence Theorem after being closely scrutinized has been found sound and well.

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Appendix A

The renormalized self-energies of the Standard Model, that we will use in the Equivalence Theorem are expressed in terms of the bare self-energies in an arbitrary gauge in the following way

$$\begin{aligned}\hat{\Sigma}_w(k^2) &= \Sigma_w(k^2) - \xi_2\Sigma_T(M^2) + \delta Z_w(k^2-\xi_2 M^2) - \xi_2 M^2\delta Z_{\xi_2},\\ \hat{\Sigma}_T(k^2) &= \Sigma_T(k^2) - \Sigma_T(M^2) + \delta Z_W(k^2-M^2),\\ \xi_1\hat{\Sigma}_L(k^2) &= \xi_1\Sigma_L(k^2) - \xi_1\Sigma_T(M^2) + \delta Z_W(k^2-\xi_1 M^2) - k^2\delta Z_{\xi_1},\\ \hat{\Sigma}_{Ww}(k^2) &= \Sigma_{Ww}(k^2) + \frac{1}{2}M(\delta Z_{\xi_1} - \delta Z_{\xi_2}).\end{aligned} \tag{A.1}$$

In the previous expression M is the renormalized W-boson mass. In the on-shell scheme the renormalization constants can be written in terms of bare self-energies

$$\begin{aligned}\delta Z_M &= \frac{1}{M^2}\Sigma_T(M^2),\\ \delta Z_{\xi_1} &= \frac{1}{M^2}\left(\Sigma_L(\xi_1 M^2) - \Sigma_T(M^2)\right),\\ \delta Z_{\xi_2} &= \frac{1}{M^2}\left(\frac{\Sigma_w(\xi_2 M^2)}{\xi_2} - \Sigma_T(M^2)\right).\end{aligned} \tag{A.2}$$

The last two relations are deduced from eq.(A.1) by imposing the on-shell condition

$$\begin{aligned}\hat{\Sigma}_w(\xi_2 M^2) &= 0,\\ \hat{\Sigma}_L(\xi_1 M^2) &= 0.\end{aligned} \tag{A.3}$$

The external renormalization constants for the W and π fields are given by

$$\begin{aligned}\tilde{Z}_W &= 1 - \frac{\partial\Sigma_T(k^2)}{\partial k^2}|_{k^2=M^2}\\ \tilde{Z}_w &= 1 - \frac{\partial\Sigma_w(k^2)}{\partial k^2}|_{k^2=\xi_2 M^2}\end{aligned} \tag{A.4}$$

They differ from the field renormalization constants Z_W, Z_w by a finite amount.

Here we will give the divergent parts and M_H dependence of the bare self-energies that are needed in order to evaluate the $\mathcal{C}$ factor in the Standard Model. In these expressions we have taken $\xi_1 = \xi_2 = \xi$. The divergent contributions are

$$\begin{aligned}\Sigma_T(k^2) &= \frac{g^2}{8\pi^2}\frac{1}{\epsilon}M^2(+\frac{3}{2} - \frac{3}{4c_w^2} - \xi(\frac{1}{4c_w^2} - \frac{1}{2})) - \frac{g^2}{8\pi^2}\frac{k^2}{\epsilon}(-\frac{25}{6} + \xi),\\ \Sigma_L(k^2) &= \frac{g^2}{8\pi^2}\frac{1}{\epsilon}M^2(+\frac{3}{2} - \frac{3}{4c_w^2} - \xi(\frac{1}{4c_w^2} - \frac{1}{2})),\\ \Sigma_w(k^2) &= \frac{g^2}{8\pi^2}\frac{1}{\epsilon}k^2(\frac{3}{2} + \frac{3}{4c_w^2} - \xi(\frac{1}{2} + \frac{1}{4c_w^2})).\end{aligned} \tag{A.6}$$

And the Higgs dependent contribution is

$$
\begin{aligned}
\Sigma_T(k^2) &= \frac{g^2}{8\pi^2} M^2 \left(-\frac{1}{16}\frac{M_H^2}{M^2} - \frac{3}{8}\ln M_H^2\right) - \frac{g^2}{8\pi^2}\frac{k^2}{24}\ln M_H^2, \\
\Sigma_L(k^2) &= \frac{g^2}{8\pi^2} M^2 \left(-\frac{1}{16}\frac{M_H^2}{M^2} - \frac{3}{8}\ln M_H^2\right), \\
\Sigma_w(k^2) &= \frac{g^2}{8\pi^2} k^2 \left(\frac{1}{16}\frac{M_H^2}{M^2} + \left(\frac{3}{8} - \frac{\xi}{4}\right)\ln M_H^2\right).
\end{aligned} \tag{A.7}
$$

Appendix B

In the on-shell scheme one usually imposes a unit residue wave function renormalization condition on the Higgs self-energy. In the effective field theory the Higgs has dissappeared and it does not seem very sensible to give renormalization conditions over a non-existing field. We will replace the condition from the Higgs self-energy to the Goldstone boson one. We will follow the discussion of renormalization conditions in the section 4 of the paper by Bohm et al[16]. The renormalization conditions in the on-shell scheme are:
(1) The propagators have poles at the physical masses of the particles

$$
\hat{\Sigma}_T^W(M^2) = 0 \qquad \hat{\Sigma}_T^Z(M_Z^2) = 0 \qquad \hat{\Sigma}_H(M_H^2) = 0 \tag{B.1}
$$

(2) The electric charge is defined as in QED implying

$$
\hat{\Sigma}_T^{\gamma Z}(0) = 0 \tag{B.2}
$$

(3) The photon and Higgs propagator have unit residue

$$
\frac{1}{k^2}\hat{\Sigma}_T^{\gamma}(k^2)|_{k^2=0} = 0 \qquad \frac{\partial}{\partial k^2}\hat{\Sigma}_H(k^2)|_{k^2=M_H^2} = 0 \tag{B.3}
$$

(4) Similar requirements are imposed on the unphysical sector

$$
\hat{\Sigma}_L^W(\xi^W M^2) = 0 \qquad \hat{\Sigma}_L^Z(\xi^Z M_Z^2) = 0 \qquad \hat{\Sigma}_{w^+}(\xi^W M^2) = 0 \qquad \hat{\Sigma}_{w^3}(\xi^Z M_Z^2) = 0
$$

$$
\hat{\Sigma}^{\gamma w^3}(0) = 0 \qquad \frac{1}{k^2}\hat{\Sigma}_L^{\gamma}(k^2)|_{k^2=0} = 0 \tag{B.4}
$$

(5) The vanishing tadpole condition

$$
v^2 + \frac{\mu^2}{\lambda} + \delta T = 0 \tag{B.5}
$$

where δT is the tadpole contribution generated in perturbation theory. In the Effective Chiral Lagrangian the requirements on the Higgs self-energy and the tadpole condition do not make sense anymore. On the other hand we have two renormalization constants less,

namely $\delta\lambda$ and $\delta\mu$. When we use the Effective Chiral Lagrangian we replace $\omega \to \pi$ and instead of the second equation of (B.3) we take

$$\frac{\partial}{\partial k^2}\hat{\Sigma}_{\pi^+}(k^2)|_{k^2=\xi^W M^2} = 0\,. \tag{B.6}$$

This fixes Z_π and we are still left with a compatible system of equations. By solving the remaining equations it is quite easy to see that the change in Z_π affects only to δv. In order to evaluate these changes one can use the first condition of eq. (B.1)

$$\frac{\delta v}{v} = \frac{1}{2}\left(-\frac{\Sigma_T^W(M^2)}{M^2} - \delta Z_W - 2\frac{\delta g}{g} + \delta Z_\pi\right). \tag{B.7}$$

Appendix C

The set of C and P and $SU(2)_L \times U(1)$ gauge invariant operators $\mathcal{L}_i$ are

$$\begin{aligned}
&\mathcal{L}_0 = \tfrac{1}{4}a_0 v^2 T_\mu T^\mu && \mathcal{L}_1 = \tfrac{1}{2}a_1 g g' B_{\mu\nu}\mathrm{Tr}TW^{\mu\nu}\\
&\mathcal{L}_2 = i a_2 g' B_{\mu\nu}\mathrm{Tr}[TV^\mu V^\nu] && \mathcal{L}_3 = -i a_3 g \mathrm{Tr}[W^{\mu\nu}[V_\mu, V_\nu]]\\
&\mathcal{L}_4 = a_4 \mathrm{Tr}[V_\mu V_\nu]\mathrm{Tr}[V^\mu V^\nu] && \mathcal{L}_5 = a_5 \mathrm{Tr}[V^\mu V_\mu]\mathrm{Tr}[V^\nu V_\nu]\\
&\mathcal{L}_6 = a_6 \mathrm{Tr}[V_\mu V_\nu]T^\mu T^\nu && \mathcal{L}_7 = a_7 \mathrm{Tr}[V_\mu V^\mu]T^\nu T_\nu\\
&\mathcal{L}_8 = -\tfrac{1}{4}a_8 g^2 \mathrm{Tr}[TW_{\mu\nu}]\mathrm{Tr}[TW^{\mu\nu}] && \mathcal{L}_9 = -i a_9 g \mathrm{Tr}[TW^{\mu\nu}]\mathrm{Tr}[TV^\mu V^\nu]\\
&\mathcal{L}_{10} = a_{10}(T_\mu T_\nu)^2 && \mathcal{L}_{11} = a_{11}\mathrm{Tr}[(\mathcal{D}_\mu V^\mu)^2]\\
&\mathcal{L}_{12} = a_{12}\mathrm{Tr}[T\mathcal{D}_\mu \mathcal{D}_\nu V^\nu]T^\mu && \mathcal{L}_{13} = \tfrac{1}{2}a_{13}(\mathrm{Tr}[T\mathcal{D}_\mu V_\nu])^2
\end{aligned} \tag{C.1}$$

where

$$\begin{aligned}
&V_\mu = (D_\mu U)U^\dagger \qquad T = U\tau_3 U^\dagger \qquad T_\mu = \mathrm{Tr}TV_\mu\\
&\mathcal{D}_\mu O(x) = \partial_\mu O(x) + ig\,[W_\mu, O(x)]
\end{aligned} \tag{C.2}$$

Expanding the operators of eq.(C.1) up to two fields one finds

$$\begin{aligned}
\Sigma_T^W(k^2) =&0 \qquad \Sigma_L^W(k^2) = -g^2 a_{11}k^2 \qquad \Sigma_\pi(k^2) = 4a_{11}\frac{1}{v^2}k^4\\
\Sigma_T^Z(k^2) =&k^2(c_w^2 g^2 a_8 + 2s_w^2 g^2 a_1 + \frac{g^2}{c_w^2}a_{13}) + 2M_Z^2 a_0\\
\Sigma_T^\gamma(k^2) =&k^2(s_w^2 g^2(a_8 - 2a_1))\\
\Sigma_T^{\gamma Z}(k^2) =&k^2\left(s_w c_w g^2 a_8 - (c_w^2 - s_w^2)gg' a_1\right)
\end{aligned} \tag{C.3}$$

Appendix D

We expand the Standard Model lagrangian in the non-linear variables in the number of fields. The hats mean that the two, three and four $\hat{\mathcal{L}}$ include the contributions from the L_{GF} that do not depen on ξ and in $\hat{\mathcal{L}}_{GF}$ it is written the ξ dependent pieces.

$$\mathcal{L} = \hat{\mathcal{L}}_2 + \hat{\mathcal{L}}_3 + \ldots + \hat{\mathcal{L}}_{GF\,2} + \hat{\mathcal{L}}_{GF\,3} + \ldots$$

$$\begin{aligned}
\hat{\mathcal{L}}_2 =& \frac{1}{2}\partial_\mu\rho\partial_\mu\rho + \frac{1}{2}\partial_\mu\pi^3\partial_\mu\pi^3 + \partial_\mu\pi^+\,\partial_\mu\pi^- - v\rho(\mu^2+\lambda v^2) - \frac{1}{2}\rho^2(\mu^2+3v^2\lambda) \\
&+ \frac{1}{4s_w^2}e^2\,v^2\,W_\mu^+\,W_\mu^- + \frac{1}{8s_w^2c_w^2}e^2\,v^2\,{Z_\mu}^2 \\
\hat{\mathcal{L}}_3 =& -\lambda v\rho^3 + \frac{1}{v}\rho\,\partial_\mu\pi^3\,\partial_\mu\pi^3 + \frac{2}{v}\rho\,\partial_\mu\pi^+\,\partial_\mu\pi^- + \frac{e}{2s_w}\,(\,\rho\,\overleftrightarrow{\partial}_\mu\,\pi^-\,)\,W_\mu^+ \\
&+ \frac{e}{2s_w}\,(\,\rho\,\overleftrightarrow{\partial}_\mu\,\pi^+\,)\,W_\mu^- - \frac{i}{2s_w}\,e\,(\,\pi^3\,\overleftrightarrow{\partial}_\mu\,\pi^-\,)\,W_\mu^+ + \frac{i}{2s_w}\,e\,(\,\pi^3\,\overleftrightarrow{\partial}_\mu\,\pi^+\,)\,W_\mu^- \\
&+ i\,e\,A_\mu\,(\,\pi^+\,\overleftrightarrow{\partial}_\mu\,\pi^-\,) + \frac{e}{2s_wc_w}\,(\,\rho\,\overleftrightarrow{\partial}_\mu\,\pi^3\,)\,Z_\mu - \frac{i}{2}\,e\,\frac{c_w^2-s_w^2}{s_wc_w}\,(\,\pi^-\,\overleftrightarrow{\partial}_\mu\,\pi^+\,)\,Z_\mu \\
&+ \frac{1}{2s_w^2}e^2\,\rho\,v\,W_\mu^+\,W_\mu^- + \frac{1}{4s_w^2c_w^2}e^2\,\rho\,v\,{Z_\mu}^2 + \frac{i}{2c_w}\,e^2\,v\,\pi^-\,W_\mu^+\,Z_\mu - \frac{i}{2c_w}\,e^2\,v\,\pi^+\,W_\mu^-\,Z_\mu \\
&- \frac{i}{2s_w}\,e^2\,v\,\pi^-\,W_\mu^+\,A_\mu + \frac{i}{2s_w}\,e^2\,v\,\pi^+\,W_\mu^-\,A_\mu
\end{aligned}$$

$$\begin{aligned}
\hat{\mathcal{L}}_4 =& -\frac{1}{4}\lambda\rho^4 + \frac{1}{2v^2}\rho^2\,\partial_\mu\pi^3\partial_\mu\pi^3 + \frac{1}{v^2}\rho^2\,\partial_\mu\pi^+\,\partial_\mu\pi^- + \frac{1}{3v^2}\partial_\mu\pi^3\,\pi^3(\partial_\mu\pi^+\,\pi^- + \partial_\mu\pi^-\,\pi^+\,) \\
&- \frac{1}{3v^2}(\partial_\mu\pi^+\partial_\mu\pi^-(\pi^+\pi^- + (\pi^3)^2) + \partial_\mu\pi^3\partial_\mu\pi^3\pi^+\pi^-) + \frac{1}{6v^2}\partial_\mu\pi^+\partial_\mu\pi^+(\pi^-)^2 \\
&+ \frac{1}{6v^2}\partial_\mu\pi^-\partial_\mu\pi^-\,(\pi^+)^2 + \frac{1}{2vs_w}e\,\rho^2\;(\partial_\mu\pi^-\,W_\mu^+ + \partial_\mu\pi^+\,W_\mu^- + \frac{1}{c_w}\partial_\mu\pi^3\,Z_\mu) \\
&+ \frac{2}{v}i\,e\,\rho\,A_\mu\,(\,\pi^+\,\overleftrightarrow{\partial}_\mu\,\pi^-) - \frac{i}{v}e\,\rho\,\frac{c_w^2-s_w^2}{s_wc_w}\,(\,\pi^-\,\overleftrightarrow{\partial}_\mu\,\pi^+\,)\,Z_\mu \\
&+ \frac{1}{4vs_w}e\,(\,2\,(\pi^-)^2\,\partial_\mu\pi^+ + 4i\,\rho\,(\,\pi^-\,\overleftrightarrow{\partial}_\mu\,\pi^3\,) + \pi^-\,\overleftrightarrow{\partial}_\mu\,(\pi^3)^2\,)\,W_\mu^+ \\
&+ \frac{1}{4vs_w}e\,(\,2\,(\pi^+)^2\,\partial_\mu\pi^- - 4i\,\rho\,(\,\pi^+\,\overleftrightarrow{\partial}_\mu\,\pi^3\,) + \pi^+\,\overleftrightarrow{\partial}_\mu\,(\pi^3)^2\,)\,W_\mu^- \\
&+ \frac{1}{2vs_wc_w}e\,(\,\pi^-\,(\pi^3\,\overleftrightarrow{\partial}_\mu\,\pi^+) + \pi^3\partial_\mu\pi^-\pi^+ + \frac{1}{2}(\pi^3)^2\partial_\mu\pi^3\,)\,Z_\mu \\
&+ \frac{1}{4s_w^2}e^2\,\rho^2\,W_\mu^+\,W_\mu^- + \frac{1}{8s_w^2c_w^2}e^2\,\rho^2\,{Z_\mu}^2 - e^2\,\pi^+\,\pi^-\,{Z_\mu}^2 + e^2\,\pi^+\,\pi^-\,{A_\mu}^2 \\
&+ \frac{1}{2}e^2\,(\,\pi^3 + 2i\rho\,)\,\pi^-\,(\,\frac{1}{c_w}Z_\mu\,W_\mu^+ - \frac{1}{s_w}\,A_\mu\,W_\mu^+\,) + e^2\,\frac{c_w^2-s_w^2}{s_wc_w}\,\pi^+\,\pi^-\,Z_\mu\,A_\mu \\
&+ \frac{1}{2}e^2\,(\,\pi^3 - 2i\rho\,)\,\pi^+\,(\,\frac{1}{c_w}Z_\mu\,W_\mu^- - \frac{1}{s_w}\,A_\mu\,W_\mu^-\,)
\end{aligned}$$

$$
\begin{aligned}
\hat{\mathcal{L}}_{GF\,2} =& -\frac{1}{2}(\frac{c_w^2}{\xi^B}+\frac{s_w^2}{\xi^W})\,(\partial_\mu A_\mu)^2-\frac{1}{2}(\frac{c_w^2}{\xi^W}+\frac{s_w^2}{\xi^B})\,(\partial_\mu Z_\mu)^2+s_w c_w\,(\frac{1}{\xi^B}-\frac{1}{\xi^W})\,\partial_\mu A_\mu\,\partial_\nu Z_\nu \\
& -\frac{1}{\xi^W}\,\partial_\mu W_\mu^+\,\partial_\nu W_\nu^- -\frac{v^2}{8}\,(g^2\xi^W+g'^2\xi^B)\,(\pi^3)^2-\frac{v^2}{4}\,g^2\xi^W\,\pi^+\pi^- \\
\hat{\mathcal{L}}_{GF\,3} =& -\frac{v}{4}\,(g^2\xi^W+g'^2\xi^B)\,\rho\,(\pi^3)^2-\frac{v}{2}\,g^2\xi^W\,\rho\,\pi^+\pi^- \\
\hat{\mathcal{L}}_{GF\,4} =& \frac{1}{24}\,(g^2\xi^W+g'^2\xi^B)\,(\pi^3)^4+\frac{1}{12}\,(2g^2\xi^W+g'^2\xi^B)\,(\pi^3)^2\,\pi^+\pi^- \\
& +\frac{g^2}{6}\xi^W\,(\pi^+)^2(\pi^-)^2-\frac{1}{8}(g^2\xi^W+g'^2\xi^B)\rho^2(\pi^3)^2-\frac{g^2}{4}\xi^W\rho^2\pi^+\pi^-
\end{aligned}
$$

The lagrangian of the Standard Model in the non-linear realization, but now in the linear gauge (2.2.12) is the following (with $\hat{\mathcal{L}}_{GF\,3,4}=0$)

$$
\begin{aligned}
\hat{\mathcal{L}}_3 =& -\lambda v\rho^3+\frac{1}{v}\rho\,\partial_\mu\pi^3\,\partial_\mu\pi^3+\frac{2}{v}\rho\,\partial_\mu\pi^+\,\partial_\mu\pi^-+\frac{e}{s_w}\,(\,\rho\,\partial_\mu\pi^-\,)\,W_\mu^+ \\
& +\frac{e}{s_w}\,(\,\rho\,\partial_\mu\pi^+\,)\,W_\mu^- -\frac{i}{2s_w}\,e\,(\,\pi^3\overleftrightarrow{\partial}_\mu\,\pi^-\,)\,W_\mu^+ +\frac{i}{2s_w}\,e\,(\,\pi^3\overleftrightarrow{\partial}_\mu\,\pi^+\,)\,W_\mu^- \\
& +i\,e\,A_\mu\,(\,\pi^+\overleftrightarrow{\partial}_\mu\,\pi^-\,)+\frac{e}{s_w c_w}\,(\,\rho\,\partial_\mu\pi^3\,)\,Z_\mu-\frac{i}{2}\,e\,\frac{c_w^2-s_w^2}{s_w c_w}\,(\,\pi^-\overleftrightarrow{\partial}_\mu\,\pi^+\,)\,Z_\mu \\
& +\frac{1}{2s_w^2}e^2\,\rho\,v\,W_\mu^+\,W_\mu^- +\frac{1}{4s_w^2c_w^2}e^2\,\rho\,v\,{Z_\mu}^2+\frac{i}{2c_w}\,e^2\,v\,\pi^-\,W_\mu^+\,Z_\mu-\frac{i}{2c_w}\,e^2\,v\,\pi^+\,W_\mu^-\,Z_\mu \\
& -\frac{i}{2s_w}\,e^2\,v\,\pi^-\,W_\mu^+\,A_\mu+\frac{i}{2s_w}\,e^2\,v\,\pi^+\,W_\mu^-\,A_\mu \\
\hat{\mathcal{L}}_4 =& -\frac{1}{4}\lambda\rho^4+\frac{1}{2v^2}\rho^2\,\partial_\mu\pi^3\partial_\mu\pi^3+\frac{1}{v^2}\rho^2\,\partial_\mu\pi^+\,\partial_\mu\pi^-+\frac{1}{3v^2}\partial_\mu\pi^3\,\pi^3(\partial_\mu\pi^+\,\pi^-\,+\partial_\mu\pi^-\,\pi^+\,) \\
& -\frac{1}{3v^2}(\partial_\mu\pi^+\partial_\mu\pi^-(\pi^+\pi^-+(\pi^3)^2)+\partial_\mu\pi^3\partial_\mu\pi^3\pi^+\pi^-)+\frac{1}{6v^2}\partial_\mu\pi^+\partial_\mu\pi^+(\pi^-)^2 \\
& +\frac{1}{6v^2}\partial_\mu\pi^-\partial_\mu\pi^-\,(\pi^+)^2+\frac{1}{2vs_w}e\,\rho^2\,\,(\partial_\mu\pi^-\,W_\mu^++\partial_\mu\pi^+\,W_\mu^-+\frac{1}{c_w}\partial_\mu\pi^3\,Z_\mu) \\
& +\frac{2}{v}i\,e\,\rho\,A_\mu\,(\,\pi^+\overleftrightarrow{\partial}_\mu\,\pi^-)-\frac{i}{v}e\,\rho\,\frac{c_w^2-s_w^2}{s_w c_w}\,(\,\pi^-\overleftrightarrow{\partial}_\mu\,\pi^+\,)\,Z_\mu+\frac{1}{3vs_w}e\,(\,\pi^-(\,\pi^-\overleftrightarrow{\partial}_\mu\,\pi^+\,) \\
& +(\pi^3+3i\rho)(\,\pi^-\overleftrightarrow{\partial}_\mu\,\pi^3\,)\,)\,W_\mu^+ +\frac{1}{3vs_w}e\,(\,\pi^+(\,\pi^+\overleftrightarrow{\partial}_\mu\,\pi^-\,) \\
& +(\pi^3-3i\rho)\,(\,\pi^+\overleftrightarrow{\partial}_\mu\,\pi^3\,)\,)\,W_\mu^- +\frac{1}{3vs_wc_w}e(\,\pi^-\,(\pi^3\overleftrightarrow{\partial}_\mu\,\pi^+)+\pi^+\,(\pi^3\overleftrightarrow{\partial}_\mu\,\pi^-))\,Z_\mu \\
& +\frac{1}{4s_w^2}e^2\,\rho^2\,W_\mu^+\,W_\mu^- +\frac{1}{8s_w^2c_w^2}e^2\,\rho^2\,{Z_\mu}^2-e^2\,\pi^+\,\pi^-\,{Z_\mu}^2+e^2\,\pi^+\,\pi^-\,{A_\mu}^2 \\
& +\frac{1}{2}e^2\,(\,\pi^3+2i\rho\,)\,\pi^-\,(\,\frac{1}{c_w}Z_\mu\,W_\mu^+-\frac{1}{s_w}\,A_\mu\,W_\mu^+\,)\,\,+e^2\,\frac{c_w^2-s_w^2}{s_w c_w}\,\pi^+\,\pi^-\,Z_\mu\,A_\mu \\
& +\frac{1}{2}e^2\,(\,\pi^3-2i\rho\,)\,\pi^+\,(\,\frac{1}{c_w}Z_\mu\,W_\mu^- -\frac{1}{s_w}\,A_\mu\,W_\mu^-\,)
\end{aligned}
$$

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Figure Captions

Fig. 1.- Tree level diagrams contributing to the scattering amplitude $A(W_L^+ W_L^- \to W_L^+ W_L^-)$ in the SM.

Fig. 2.- Tree level diagrams contributing to the Goldstone scattering amplitude $A(\omega^+\omega^- \to \omega^+\omega^-)$ in the SM.

Fig. 3.- Tree level diagrams contributing to the first next to leading correction $A(\tilde{W}^+\omega^- \to \omega^+\omega^-)$. The external gauge line is contracted not with a ϵ_L^μ but with a v_μ. To get the complete $A(\tilde{W}\omega\omega\omega)$ one should consider all permutations.

Fig. 4.- Comparison between the exact tree level amplitude of W_L's in the SM (solid line) for three different angles $\theta = \pi/4, \pi/16, 3\pi/4$ ((a),(b),(c), respectively) and four different approximations: i) the standard one, i.e., $A(\omega^+\omega^- \to \omega^+\omega^-)$ with g and g' set to zero (dashed line). ii) The complete Goldstone amplitude including Z, γ interchange diagrams (dashed-dotted line). iii) The complete $\mathcal{O}(g^2)$ contribution, i.e. ii) plus the contribution coming from the diagrams (a) (d) of Fig.3 which are of $\mathcal{O}(g^2)$ (long-dashed line, nearly invisible because overlaps the exact result). iv) In addition we have plotted in (b) an extra line (dotted) which differs from iii) in that all the denominators are expanded up to $\mathcal{O}(M^2/s, M^2/t)$ except for the Higgs propagator structure that has been kept intact.

Fig. 5.- Contribution of the $\mathcal{L}_{11}$ operator to the right hand side of the E.T. (leading and next to leading amplitude).

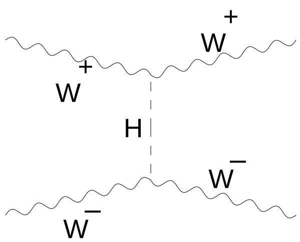
Fig. 6.- Tree level diagrams contributing to the scattering amplitude $A(W_L^+ W_L^- \to W_L^+ W_L^-)$ in an Effective Chiral Lagrangian up to $\mathcal{O}(p^4)$. The black circle means that the vertex includes contributions from $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ operators and the cross means that there are only contributions from $\mathcal{O}(p^4)$ operators.

Fig. 7.- (a)-(e) are the tree level diagrams contributing to the scattering amplitude $A(\pi^+\pi^- \to \pi^+\pi^-)$ in an Effective Chiral Lagrangian up to $\mathcal{O}(p^4)$. (f) + permutations are the first contributions coming from the next to leading amplitude $A(\tilde{W}\pi\pi\pi)$.

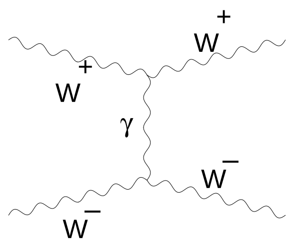
Fig. 8.- Comparison between the exact tree level amplitude of W_L in an Effective Chiral Lagrangian for the particular a_i's that corresponds to the SM, i.e. $a_5^{tree} = v^2/8M_H^2$ and the rest of a_i set to zero (same conventions for the lines as in Fig.4).

Fig. 9.- Comparison between the standard approach done in the literature $A(\pi^+\pi^- \to \pi^+\pi^-)$ with $g = g' = 0$ (dashed line) in front of the exact result and our approach (long-dashed line) for (a) $a_5 = v^2/8M_H^2$ and (b) $a_5 = 16/384\pi^2$. θ is $\pi/5$.

Fig.1



(a)



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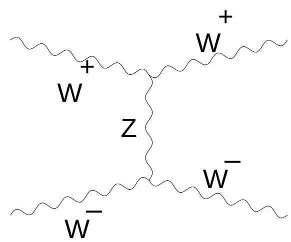

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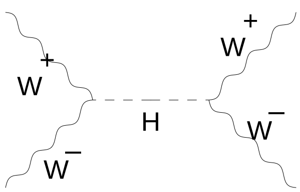

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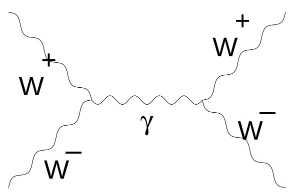
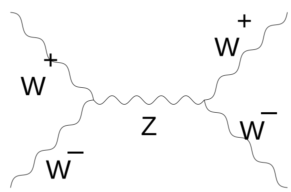
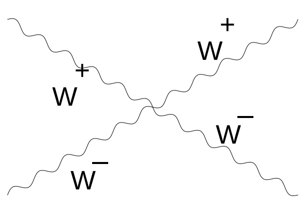


(e)



(f)



(g)

Fig.2

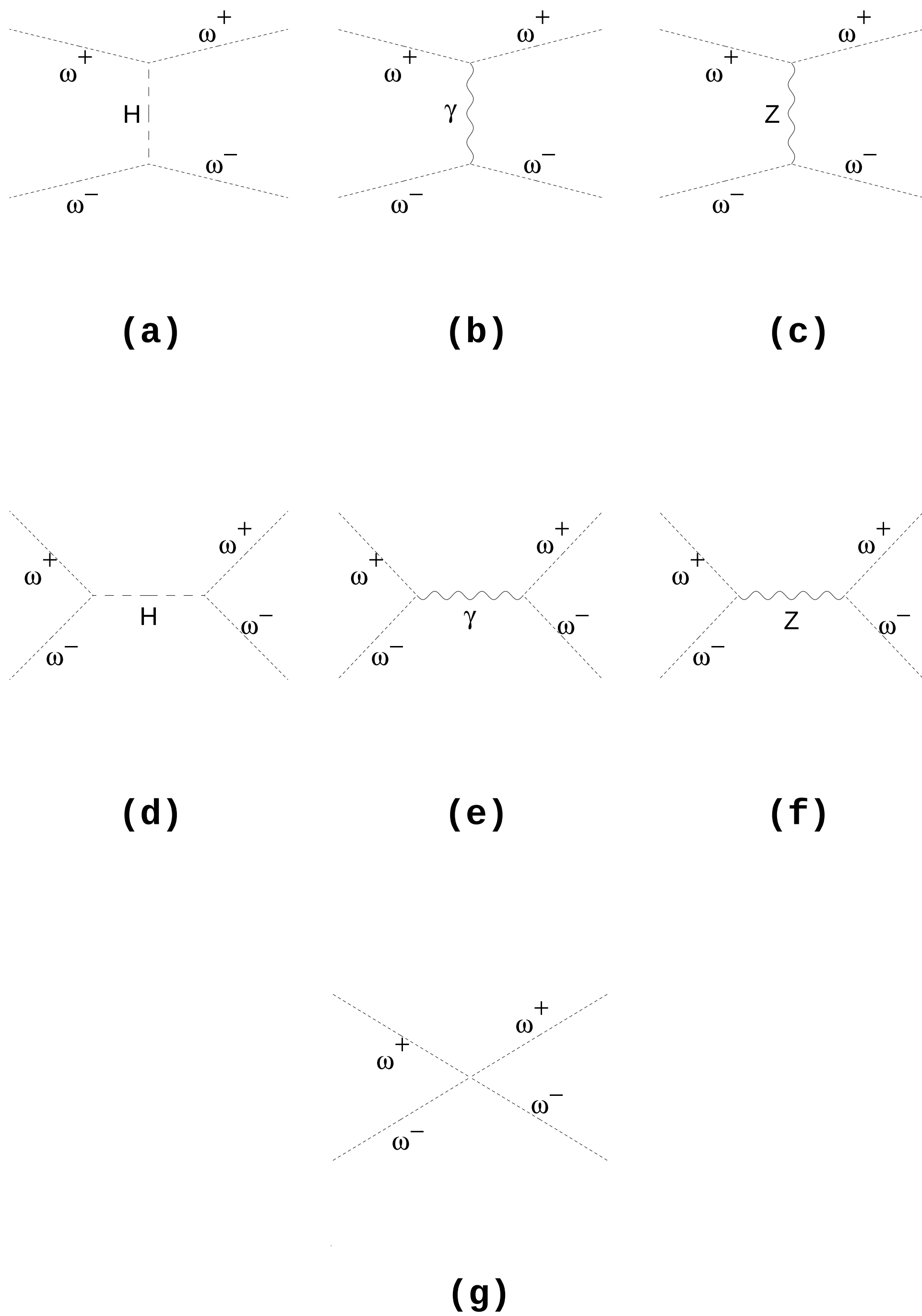


Fig.3

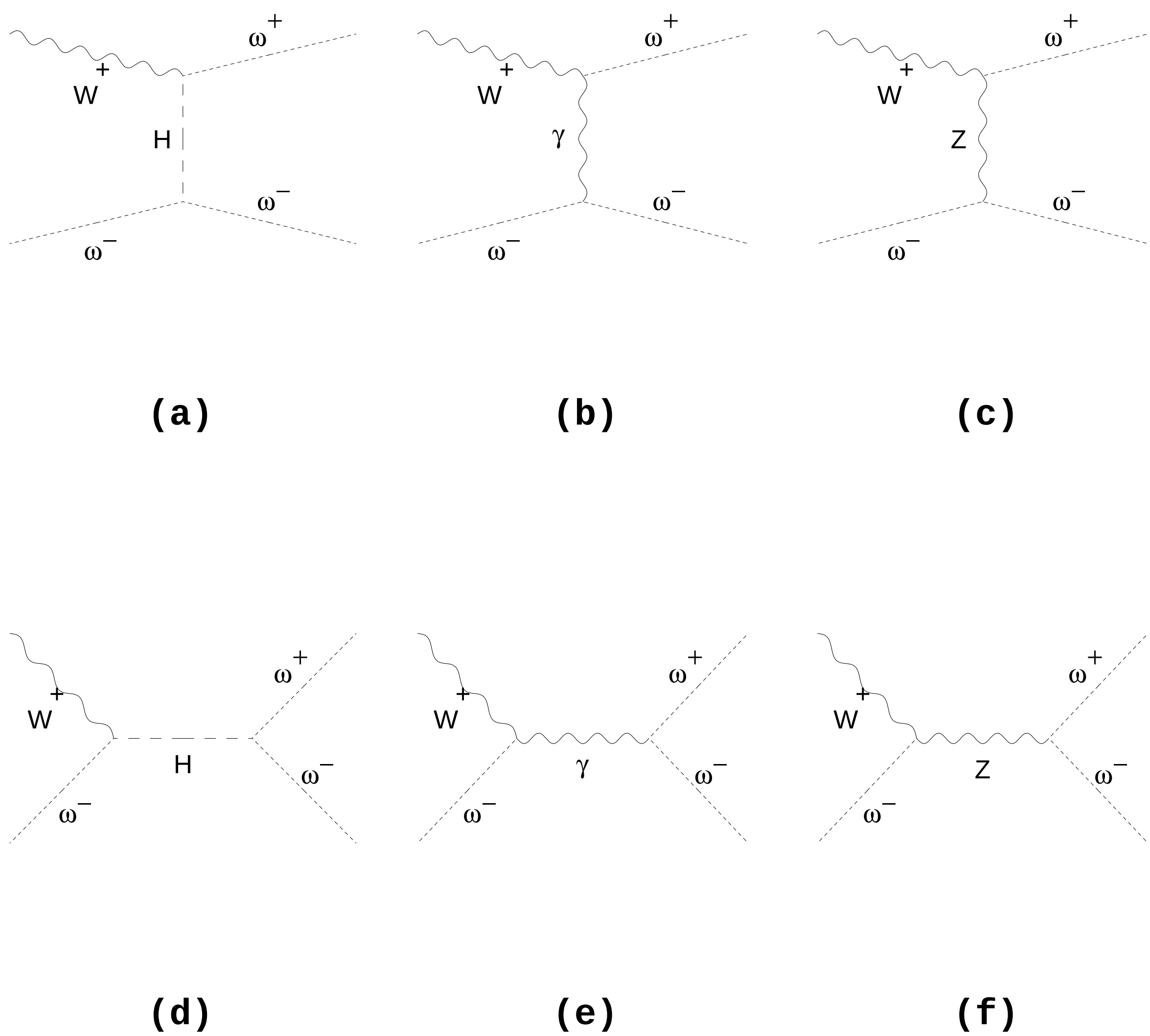


Fig.4

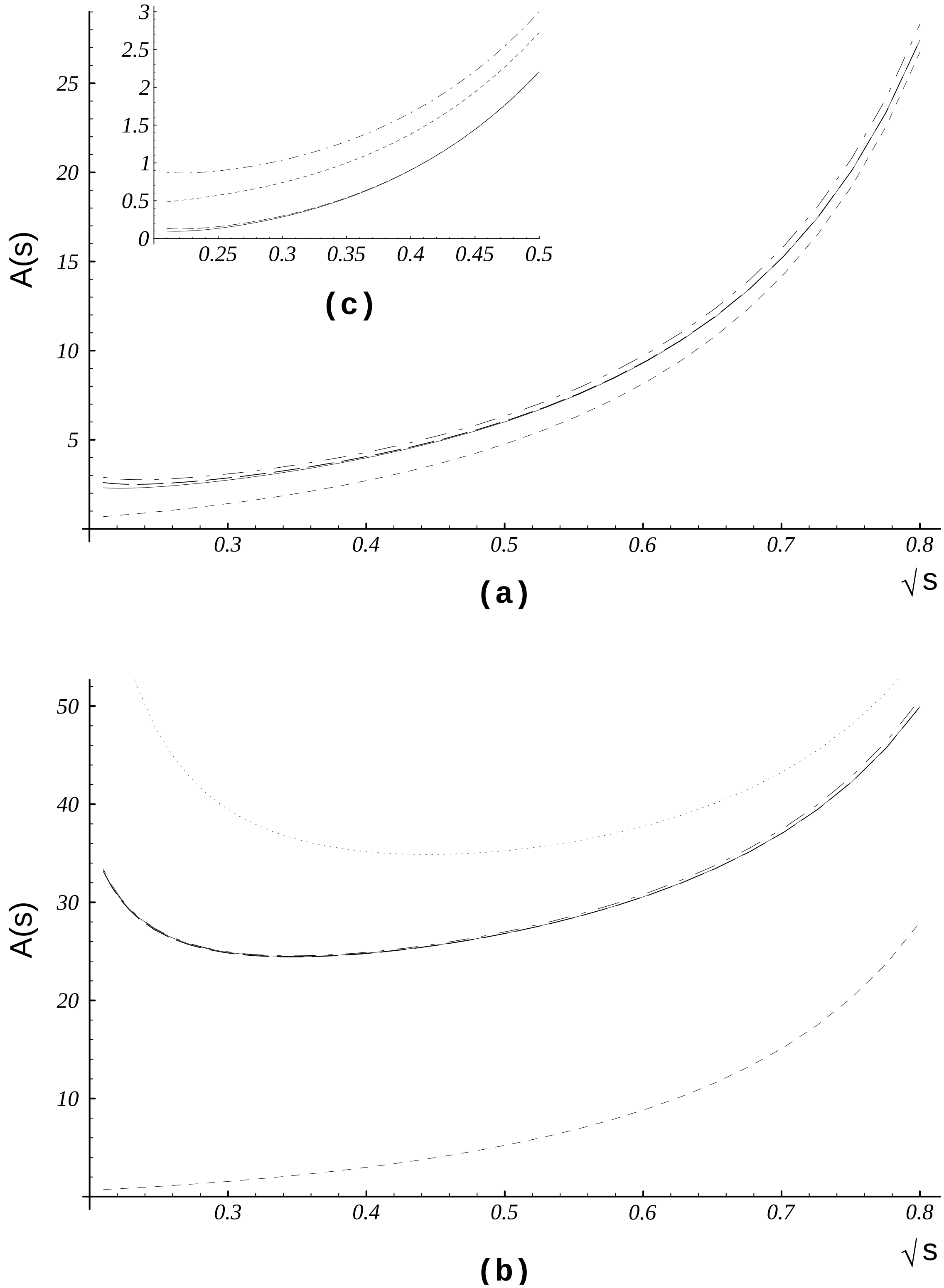
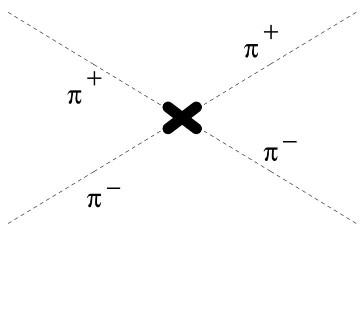
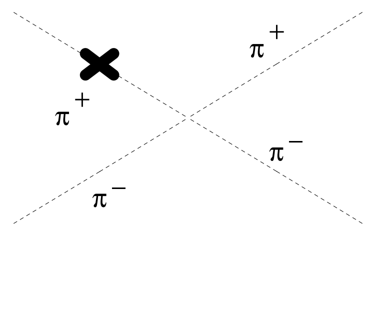

A(s)
25
20
15
10
5
0.3
0.4
0.5
0.6
0.7
0.8
√s
3
2.5
2
1.5
1
0.5
0
0.25
0.3
0.35
0.4
0.45
0.5
(c)
(a)
A(s)
50
40
30
20
10
0.3
0.4
0.5
0.6
0.7
0.8
√s
(b)

Fig.5



(a)



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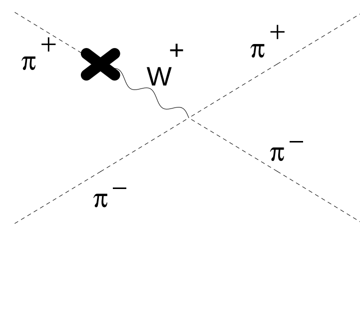
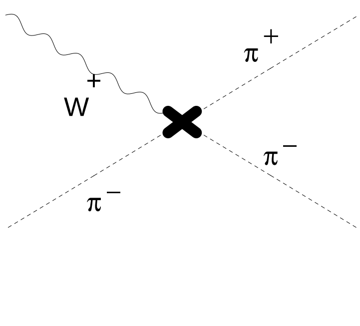
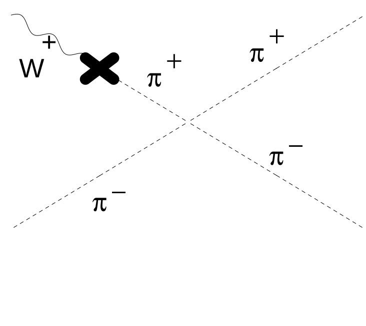


(c)



(d)



(e)

Fig.6

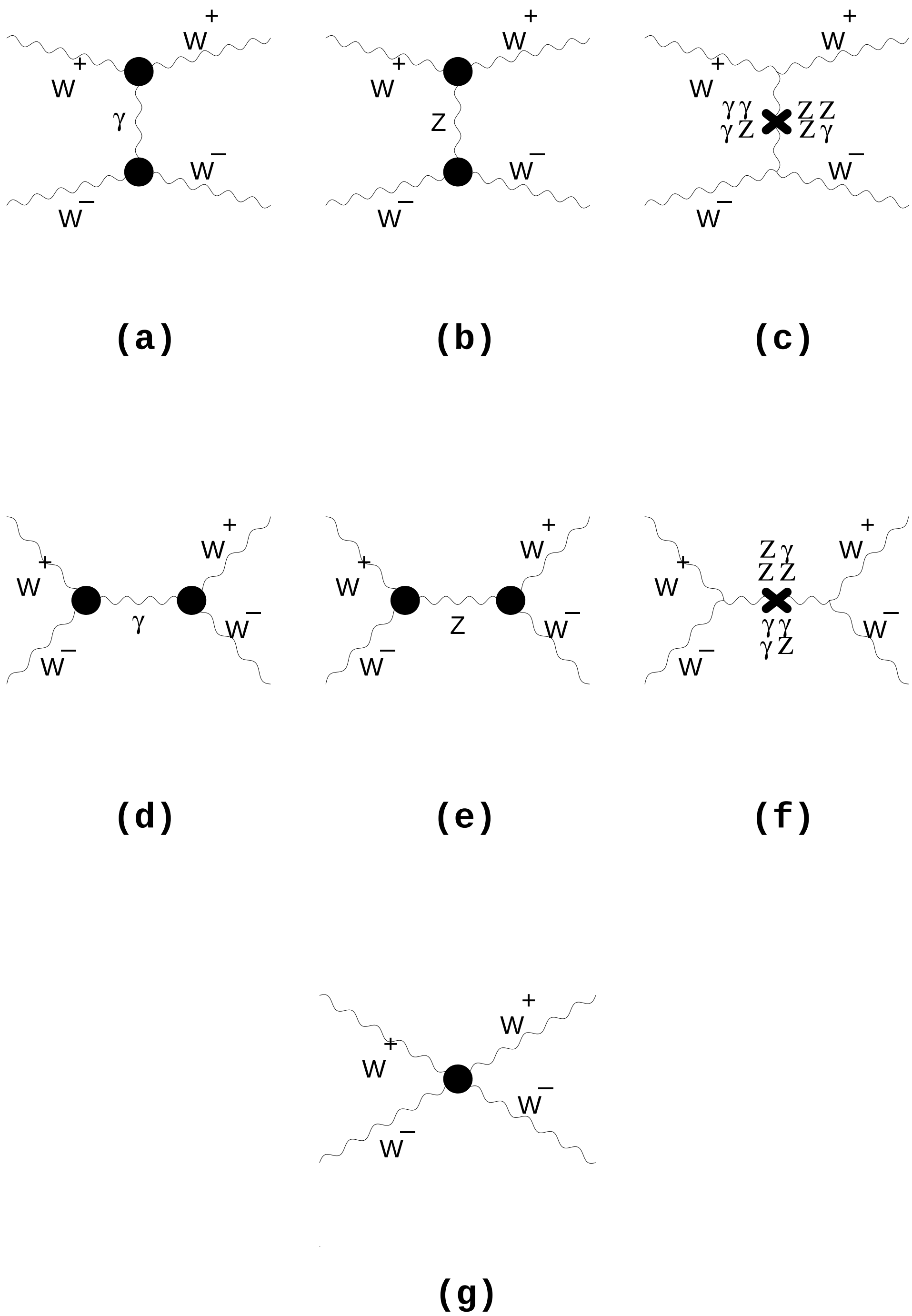
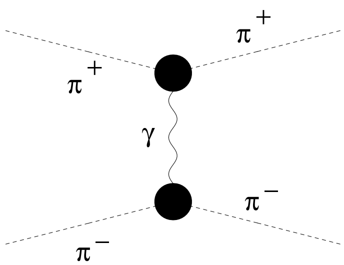
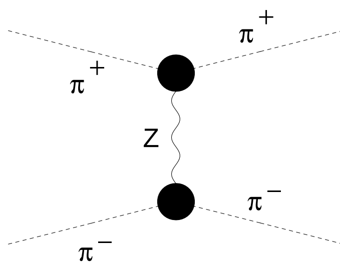


Fig.7



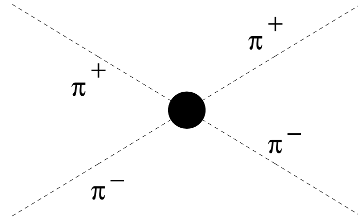
π^+
π^+
γ
π^-
π^-

(a)



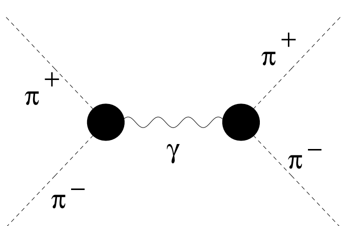
π^+
π^+
Z
π^-
π^-

(b)



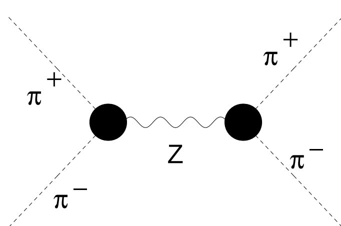
π^+
π^+
π^-
π^-

(c)



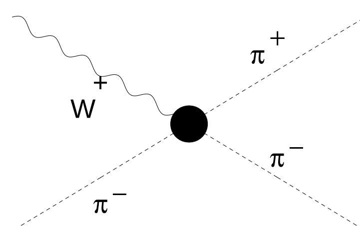
π^+
π^+
γ
π^-
π^-

(d)



π^+
π^+
Z
π^-
π^-

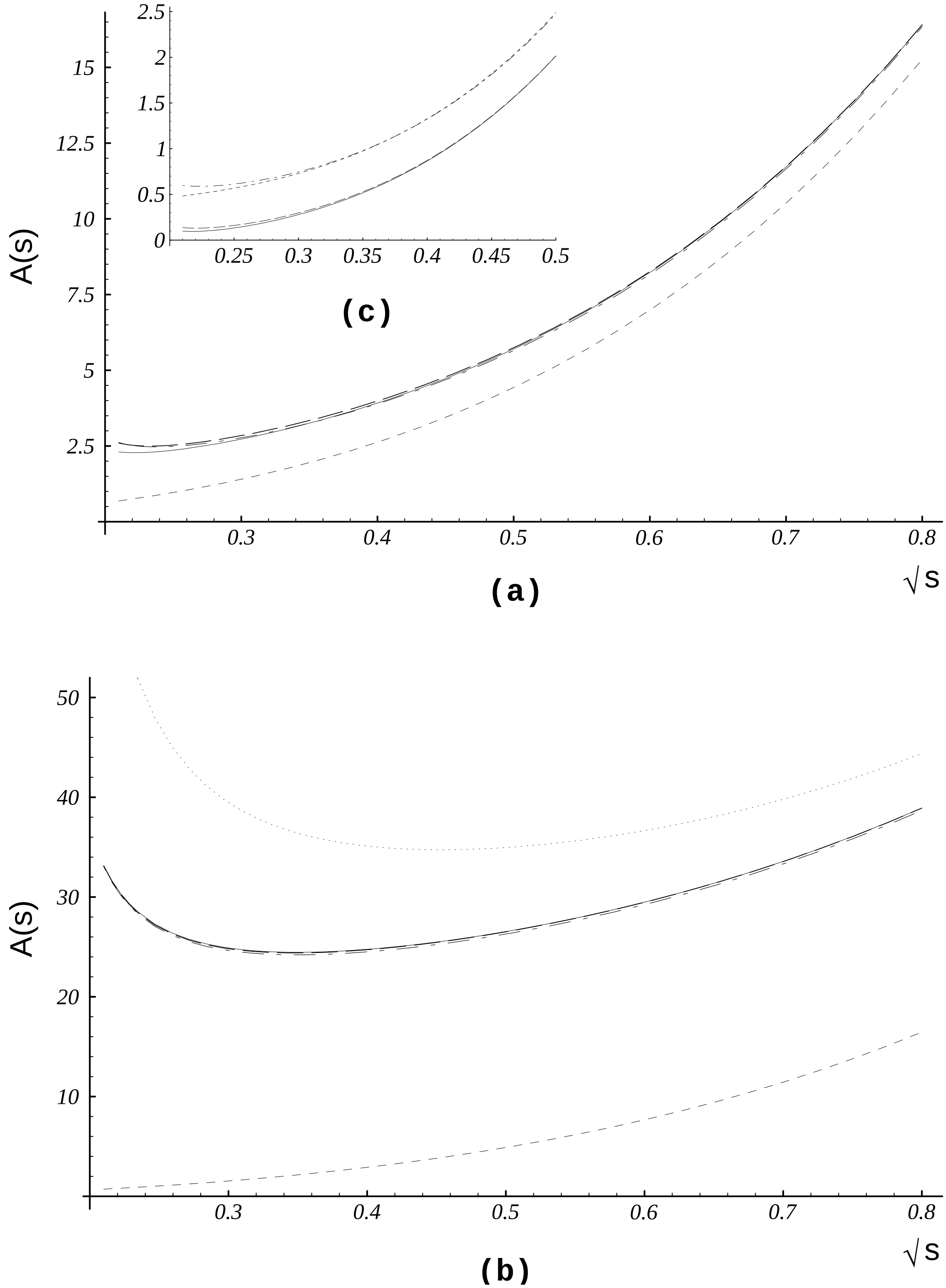
(e)



W^+
π^+
π^-
π^-

(f)

Fig.8



A(s)
15
12.5
10
7.5
5
2.5
0.3
0.4
0.5
0.6
0.7
0.8
√s
2.5
2
1.5
1
0.5
0
0.25
0.3
0.35
0.4
0.45
0.5
(c)
(a)
A(s)
50
40
30
20
10
0.3
0.4
0.5
0.6
0.7
0.8
√s
(b)

Fig.9